Improved Oil Production and Waterflood Performance by Water Allocation Management

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ABSTRACT

This thesis has evaluated a wide range of techniques to militate one of the big challenges of petroleum industry, water production. The techniques discussed waterflooding management, all aim to reduce excessive water production. The injection and production history at a well and field level are most common available data in any oil field, especially when nowadays we can have these data in real time with the implementation of the digital oil field and the intelligent well completion. This research aims to understand the strength and weaknesses of the existing techniques and “repackage” them to provide an optimum combination for more effective waterflood management by analysing injection and production data history.

The first part of this research reviewed, tested and compared the analytical techniques that have been previously used for analysing the injection and production. The methods studied fell in to two distinct classes: those that monitor the waterflood performance secondly, methods for determining the inter-well connectivity.

The second part of this thesis showed that an improved workflow used the captured information from the phase one methods could be combined to give more effective waterflood management via combination of reservoir voidage management (*RVM*), water allocation management (*WAM*) and production allocation management (*PAM*).

Finally, a semi-analytical method was introduced in this thesis for performing *RVM*. Two approaches were defined for *WAM* and new techniques developed for *PAM*, all of which employed only the production and injection history. The results from these techniques were compared with the more advanced reservoir simulation methodologies such as gradient free optimisation. This comparison showed the reliability of the proposed techniques.

DEDICATION

I dedicate this thesis to my wife and my parents for their continuous encouragement and love.

Acknowledgement

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AF	allocation factor
ANNs	Artificial Neural Networks
BHP	bottom hole pressure
CANFIS	Co-adoptive neuro-fuzzy inference system
COC	Cumulative oil cut
COR	Cumulative oil ratio
CRM	capacitive resistive mode
CRMIP	Applied CRM for drainage volume between each injector-producer pair
CRMP	Applied CRM for drainage volume of each producer
CRMT	Applied CRM for volume of the entire field or tank model
CWC	Cumulative water cut
FFBP	Feed-forward back propagation network
GA	Genetic algorithm
ICV	inflow control valves
IWC	Intelligent Well Completion
LP	Linear programming
LR	liquid production ratio
MLR	Multi-Linear Regression
OI	oil production performance index
OOIP	original oil in place
OPI	oil production index
OPR	Relative oil production ratio
OR	Oil Ratio
PAM	Production allocation management
RVM	Reservoir voidage management
PE	Processing elements
VR	Voidage ratio
WAM	water allocation management
WC	water cut
WFM	Waterflood management
WOR	Water oil ratio
WR	Water ratio

NOMENCLATURE

β_{ij}	MLR weighting factors
β_{0j}	unbalance constant for unbalanced *MLR*
Z	transfer function
V_{pr}	reservoir pore volume
V_p	drainage pore volume
t_{pm}	time of starting pressure maintenance by water injection
t_2	the duration time of injection with the second *VR*
t	production time
S_{wc}	connate water saturation
S_w	water saturation
S_{av}	average water saturation
r_w	well-bore radius
r_s	Spearman rank correlation coefficient
r_e	well influence zone radius
Q_{wr}	reservoir's cumulative water production
Q_{wibt2}	injected cumulative water volume before water break through by *VR2*
Q_{wibt1}	injected cumulative water volume before water break through by *VR1*
Q_{wibt}	injected cumulative water volume before water break through
Q_w	cumulative water production
Q_{or}	reservoir's cumulative oil production
Q_o	cumulative oil production
q_l	liquid production rate
q_l	liquid production rate
Q_l	cumulative liquid production
q_i	injection rate
q_i	injection rate
p_{wf}	flowing bottom hole pressure
P_{rmin}	minimum reservoir pressure
P_r	initial reservoir pressure
P_i	injection pressure
P_e	reservoir pressure
PAF_j	production allocation factor for the producer *j*

OR_j	oil ratio for the producer j
OPI_{rj}	difference between producer OPI_j and reservoir OPI_r
n	number of observations
N	total number of producers
k	formation permeability
J	productivity index
I	total number of injectors
II	Injectivity index
H	formation thickness
f_w	fractional flow of water
f_{ij}	CRM inter-well connectivity between injector I and producer j
E_R	Oil recovery
dV	change in reservoir volume
dP	change in reservoir pressure
d_i	difference between the rankings of the ith observations
C_t	Reservoir total compressibility
C	compressibility factor
Bo	Oil formation volume factor
AF_{ti}	total water allocation index for injector i
AF_{ni}	normalized allocation factor for the injector i
a and b	permeability ratio constants
μ	injected fluid viscosity
$\frac{k_{rw}}{k_{ro}}$	relative permeability ratio of water to oil
$\frac{\mu_o}{\mu_w}$	viscosity ratio of oil to water
$q_j(t)$	production rate of producer j
$i_i(t)$	injection rate of injector i
$\bar{q}_j$	average production rate
$\bar{i}_i$	average injection rate
μ_j	Lagrange multiplier
$\bar{P}$	average reservoir pressure

τ	CRM time constant
I_{ij}^{k}	injection rate of injector *i* at time interval k in *CRM* equation
$\Delta P_{wf,j}^{(k)}$	changes in *BHP* of producer *j* during the time interval t_{k-1} to t_{k}
AF_{ni}^{+}	production wells with positive AF_{ni}
AF_{ni}^{-}	production wells with negative AF_{ni}
OPR_{j}^{-}	wells with negative *OPR*
OPR_{j}^{+}	wells with Positive *OPR*
OPI_{rj}^{+}	wells with Positive OPI_{rj}
OPI_{rj}^{-}	wells with negative OPI_{rj}
q_{oj}^{-}	old allocated production rate correspond to wells with negative *OPR* or *OPI*
q_{nj}^{-}	new allocated production rate correspond to wells with negative *OPR* or *OPI*
q_{oj}^{+}	old allocated production rate correspond to wells with positive *OPR* or *OPI*
q_{nj}^{+}	new allocated production rate correspond to wells with positive *OPR* or *OPI*

Chapter 1– Introduction

Primary recovery methods using natural flow mechanisms, rock liquid and gas expansion plus solution gas drive leave 80% or more of the original oil in place after the pressure in the reservoir has been depleted. A vast amount of hydrocarbon thus remains unrecovered in the world's depleted reservoirs. Billions barrels of additional oil could have been recovered through waterflooding, the most important method for improving recovery from oil reservoirs [1].

Water flooding is often initiated after reservoir (pressure) depletion has occurred. It is frequently the case that (free) gas saturation, or gas cap, forms in a solution gas drive reservoir due to this pressure depletion. Initially, the increasing reservoir pressure due to the injected water will force the free gas to redissolve into the oil. The response of the oil production rate to the water injection will become most apparent after the water injection has achieved fill-up of the gas space (Figure 1.1) [1].

The maximum oil production rate as the reservoir is re-pressurised depends upon the reservoir characteristics and the injection rate. Continued water injection will eventually lead to "breakthrough" of the injected water at the production wells. This peak oil production rate will then start to decline as the water cut increases [1].

1.1 Water production

One of the most significant challenges in oil production operations relates to the production of water. This excessive water production is recognised not only as a large operational problem, but is also a significant economic and environmental one [2]. This is particularly relevant when discussing mature fields, where water can be continuously pumped into a reservoir with little oil being produced [3]. Many of these mature fields in decline are currently producing at well above 80% water cut. A scenario such as this has several negative implications, one of which being that the majority of the energy supplied to the artificial lift system installed in the production wells will be spent on lifting water, rather than oil [4]. More importantly, government regulations regarding the treatment and disposal of this water are continually becoming more rigorous, and trends in the development of legislation suggest that meeting legal requirements will require increasingly costly water treatments [3, 5]. Estimates for global water production vary and are generally hard to quantify. What can easily be identified however is the growing trend in the water to oil ratio in terms of total global production. In the mid-1990s, the water to oil ratio in North America was estimated at approximately 7:1 [6], increasing to almost 10:1 ten years later [7]. Extrapolation of this trend, suggests that the ratio of produced water to oil is currently well in excess of 10:1 [2]. The costs of treating this water have been estimated to be in excess of $40-50 billion per year, implying that development of new options to reduce this amount has become a top priority within the industry [6].

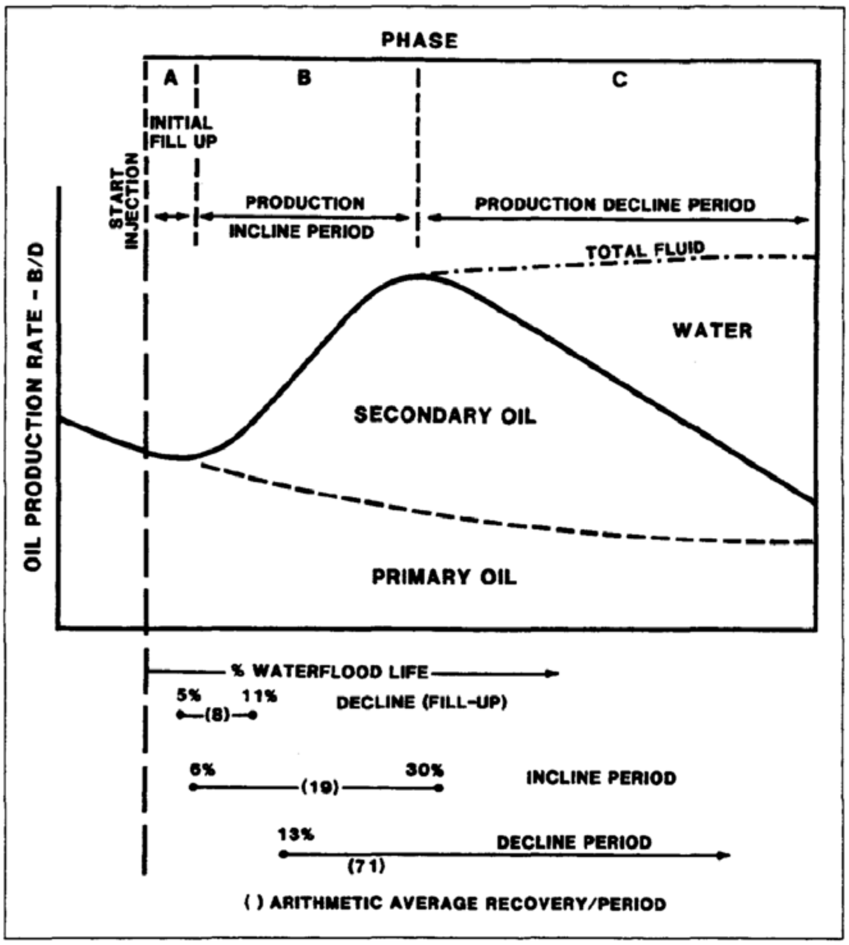

Figure 1. 1: Waterflood performance at different periods [8].

There are, of course, other risks associated with water production that stretch beyond the handling and processing costs. Scaling, corrosion and reservoir souring are all significant challenges facing engineers around the world today [9]. These scenarios can be only remediated by well workovers or remedial treatments, so these unnecessary situations have significant impacts both technically, through lost performance, and also financially, in lost revenue from decreased or halted production. Poor sweep efficiency of water injection is another challenge that frequently requires attention. This unfavourable situation can lead to bypassed oil, as well as unexpected and undesirable early water breakthrough [5]. The problem has not been devoted to attention. Finding approaches to dealing with these issues has been recognised as an effective way to increase the overall recovery efficiency in a cost effective manner [2].

1.2 Waterflooding management

A review of the underlying physics of the water flood recovery processes indicates that their efficiency is influenced by:

1. Timing of the initiation of the waterflood
2. Rate of water injection into and fluid production from the reservoir
3. Location of the water injection and distribution of water between injectors

Improper injection may cause early water breakthrough, higher water production and lower recovery, leading to the unnecessary cost of handling produced water. Two important questions need be answered for efficient waterflood performance.

1. What is the optimal total amount of water needed to be injected into the reservoir?
2. How should be the water be allocated to the different injectors?

Water injection has two roles; (i) to maintain the reservoir pressure at a suitable level and (ii) to push the oil toward the producers. The first question requires development of the pressure maintenance plan, once the reservoir engineers defined the preferred reservoir pressure. This pressure should ideally support the flow of fluid from the reservoir to the producers and from bottom of the well to the surface, otherwise artificial lift techniques will need to be installed in the production wells. However, although injection of a large volume of water may keep the reservoir pressure constant, it will also encourage early breakthrough of water in the producers, with the potential result of reduction in the total field oil production. Therefore, the optimum total volume of water should support the reservoir pressure, while being injected in such a manner that it delays the breakthrough time as much as possible.

The answer to the second question involves the determination of the optimum injection rate allocation process for the injection wells. The water injection allocation management process should provide reduced support to production wells with the greatest water cut, by injecting less water into those injection wells supporting these bad producers.

1.3 Intelligent Well Completion (*IWC*) and waterflooding management

With a global decline in oil production, it becomes more important to improve and increase the recovery of the remaining oil reserves. In recent years Intelligent Well Completion (*IWC*) has been employed as a possible solution to some of these associated challenges [2].

An Intelligent Well Completion (*IWC*) is essentially defined as any completion containing at least one of the following elements: multiple downhole pressure and temperature sensors, downhole flow measurement, fixed or adjustable inflow control valves (*ICV*), or a multilateral well with inflow control of one or more laterals [10]. The goal of *IWCs* has been to provide the user with one or more of the following elements: well performance monitoring, zonal productivity control, and subsequent well production optimisation (Figure 1.2) [11].

IWCs can be employed for closed loop reservoir management. The process of reservoir management by intelligent completion (Figure 1.3) can be divided into two important stages:

1) Monitoring:
    a) Data gathering by different types of gauges and sensors inside the well.
    b) Processing and interpreting the data to capture suitable desired information about the performance of the production system.
2) Controlling:
    a) Defining new production and injection strategies and scenarios.

b) Applying new schemes via control devices inside the well.

IWC can therefore be a good tool for monitoring the performance of waterflood, and then controlling the injection and production schemes to improve the efficiency of the water injection project.

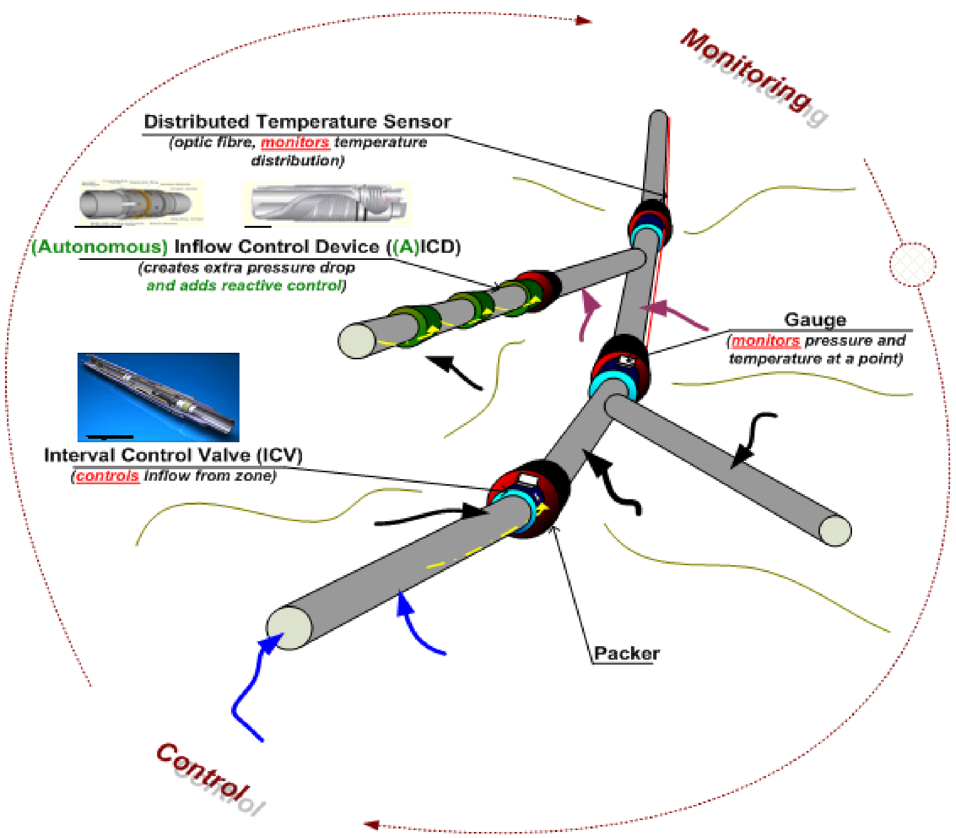

Figure 1. 2: Different components of an intelligent well completion.

One of the most important tasks in *IWC* research is how to analyse and interpret the raw data in order to capture useful information. As an example, new technique has been developed to detect the time and the location of water influx into the well [12] or zonal flow rates can be determined by analysing temperature sensor data[12, 13]. Another important challenge is developing new algorithms based on this information to control injection and production rate. This means that if we obtain good information describing the performance of the system then how should we change the system parameters to improve the performance of the system?

In this research we are interested in developing new techniques for obtaining the necessary information from injection and production history from the gauges, in order to understand waterflood performance and then, based on the captured information, managing water and production rate to improve the flooding efficiency.

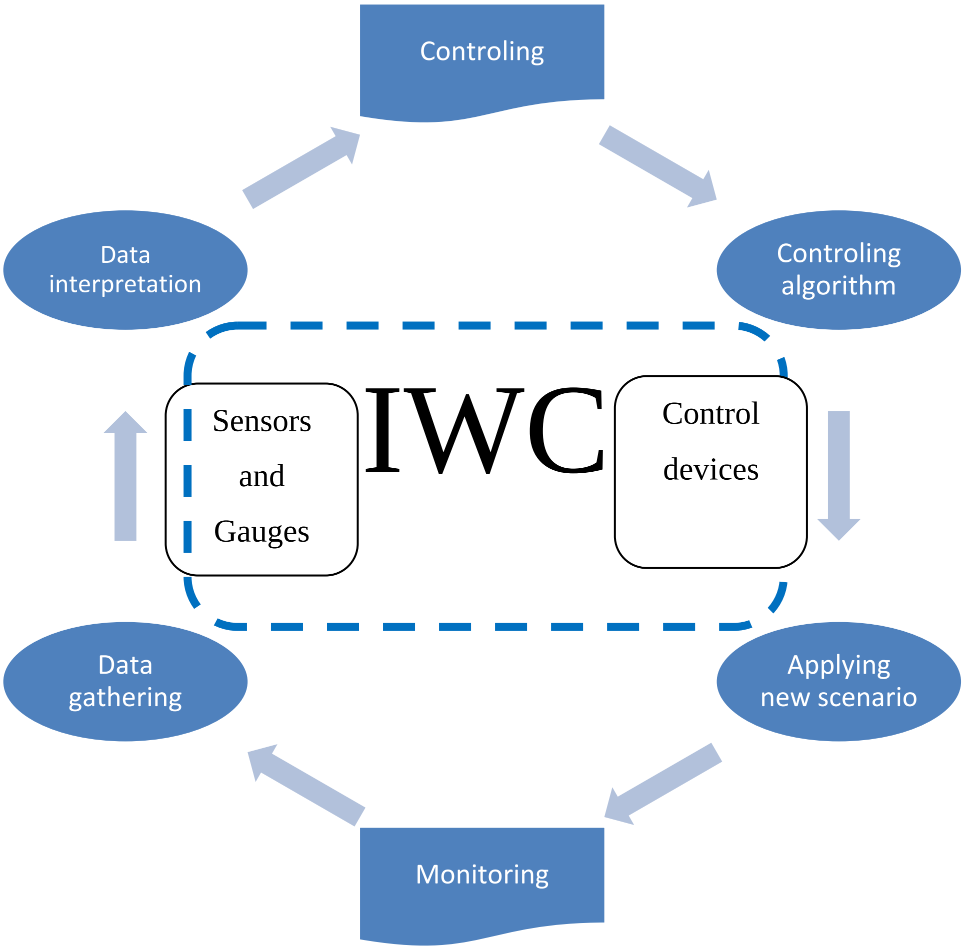

Figure 1. 3: Closed loop reservoir management by IWC.

1.4 Research objectives

The overall objective of this research is to determine the optimum amount of injected water into the reservoir and its optimal allocation between different injectors, by employing injection and production history.

In Chapter 2 will review different waterflood performance monitoring techniques to evaluate their effectiveness for monitoring the waterflood and determining performance parameters. A new technique also is proposed to control total water injection volume into the reservoir.

Two main approaches have been used in this study, to measure the optimum water allocation factor. (i) Allocation management, based on inter-well connectivity between injection wells and producers, together with performance evaluation of the production wells and (ii) The optimum water allocation factor, determined by monitoring the effect of change in the injection rate on future oil production from the production wells. Chapters 3, 4 and 5 are concerned with the first methodology. Chapter 3, reviews several statistical techniques used for inter-well connectivity measurement. The application of some artificial intelligence techniques for determining the connection between production and injection wells is examined in Chapter 4. New parameters are introduced to describe production well performance in Chapter 5. In Chapter 6, a second methodology for water allocation management, linear programming coupled with a capacitive resistive model for water allocation management, is introduced and its results compared with a combination of Genetic algorithm and reservoir simulation. Chapter 7 summarises the results of the previous chapters and discuss conclusions from the work performed.

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Chapter 2- Monitoring The Performance Of Waterflooding

2.1 Monitoring and surveillance of water injection projects

A billion barrels of additional reserves have been produced through waterflooding, one of the most important methods of improving recovery from oil reservoirs; waterflood projects account for over half of the Canadian and U.S. oil production [1]. With the economic uncertainty of EOR techniques as a result of oil-price instability, optimization and management of waterflood projects has become more important than ever [2].

A successful waterflood depends on the proper operation of individual wells in a pattern, to maintain the balance between water injection and production over the entire life of the field, and also on preventing well failures. Therefore it is necessary to design a system of monitoring and control, which acquires and processes waterflood data, and helps field engineers to make optimal decisions [3].

An essential element of a successful waterflooding project is a well-planned and well-executed program of surveillance and monitoring [4]. Surveillance and monitoring principles are the key to understanding reservoir performance and identifying opportunities that will improve ultimate oil recovery. The key ingredients of any surveillance program are planning and accurate data collection. Surveillance techniques should always be a precursor to in-depth studies, including numerical simulation [5].

A fundamental geological/petrophysical analysis is a cornerstone of good reservoir engineering analysis. However, geological studies alone do not conclusively quantify the reserve and oil rate increases that can be achieved by optimizing the existing water-floods. While a geological/ petrophysical study is key in understanding the initial question: What is the OOIP?[6] , production and pressure surveillance data can implicitly account for a useful scale of heterogeneity. Therefore, if used properly, this data can be extremely useful in developing changes in operational strategy that can maximize recovery [6].

Production, injection, water-supply, and water-disposal are four types of wells requiring surveillance. Of these, production and injection wells require the most attention;

Monitoring well performance requires a program of selected well tests to be conducted regularly [4]. The surveillance of production data is fundamental to good reservoir management of waterfloods and miscible floods. This type of surveillance is useful to understand flood performance to date and can highlight good versus poor recovery areas. In particular, surveillance can identify areas of extreme water cycling, patterns with poor sweep, or local voidage imbalances, providing “real-time” monitoring of a flood without having to construct a detailed flow-simulation model [7].

Monitoring performance of waterflooding is an important issue in the management of water injection. When sufficient data are available and production is declining, the past production curve of individual wells field can be extended to indicate future performance. In the following section we are going to review some of the common techniques employed for monitoring the performance of water injection.

2.1.1 Semi-log plot of oil cut or water cut versus cumulative oil production (cut-cum plot)

One of the commonly used performance curve analysis methods for a waterflood project is plot of water cut or oil cut versus cumulative production (Q_o) (Figure 2.1). This curve is used when the economic production rate is dictated by the cost of water disposal. A straight line extrapolation of log of water cut versus cumulative oil production may not be reasonably done in the higher water cut level. On the other hand, if oil cut data are used instead of water cut in the same levels, straight extrapolation of log of oil cut versus cumulative oil production may deteriorate and lead to optimistic reserve estimates [2, 8].

2.1.2 Semi-log plot of Water oil ratio (WOR) versus cumulative oil production

A common practice in petroleum engineering [9] has been to analyse the performance of waterfloods by plotting $log_{10}(WOR)$ versus cumulative produced oil (Q_o) for individual wells or from field data.

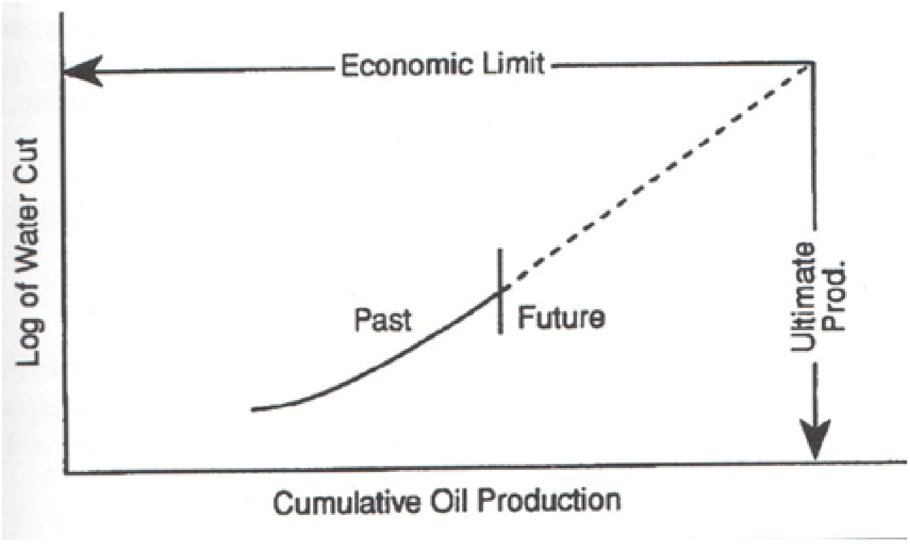


Figure 2. 1: An example of the plot of water cut versus cumulative oil production [8].

Defining the relative permeability ratio of water to oil ($\frac{k_{rw}}{k_{ro}}$) with water saturation *(S_w)* by the following equation [10, 11]:

$$\frac{k_{rw}}{k_{ro}} = ae^{bS_w} \qquad \text{(Equation 2.1)}$$

Based on the Buckley-Leverett theory for a linear, immiscible displacement, the logarithm of the water-oil ratio (*WOR*) is linearly proportional to the cumulative oil production (Q_o) .[12]

$$L\text{og}_{10}(WOR) = \left[\frac{b(1-S_{wc})}{OOIP}\right]Q_o + L\text{og}_{10}\left(\frac{a\mu_o}{\mu_w}\right) + bS_w - \frac{1}{Ln10} \qquad \text{(Equation 2.2)}$$

That is, a semi-log straight-line relationship exists between *WOR* and the cumulative produced oil, Q_o. The slope of the straight line is $\left[\frac{b(1-S_{wc})}{OOIP}\right]$ and the intercept is $L\text{og}_{10}\left(\frac{a\mu_o}{\mu_w}\right) + bS_w - \frac{1}{Ln10}$. In Equation 2.1 $\frac{\mu_o}{\mu_w}$ is the viscosity ratio of oil to water; S_{wc} is the connate water saturation; and *OOIP* is the original oil in place. The constants *a* and *b* in Equation 2.2 are derived from defining the relative permeability ratio of water to oil ($\frac{k_{rw}}{k_{ro}}$) with water saturation S_w, as defined by Equation 2.1. Therefore by plotting $log_{10}(WOR)$ versus cumulative produced oil (Q_o) for individual wells or from field data; the data generally form a straight line and this straight line can be extrapolated to predict future performance and estimate ultimate oil recovery from waterflooding (Figure 2.2). [12]

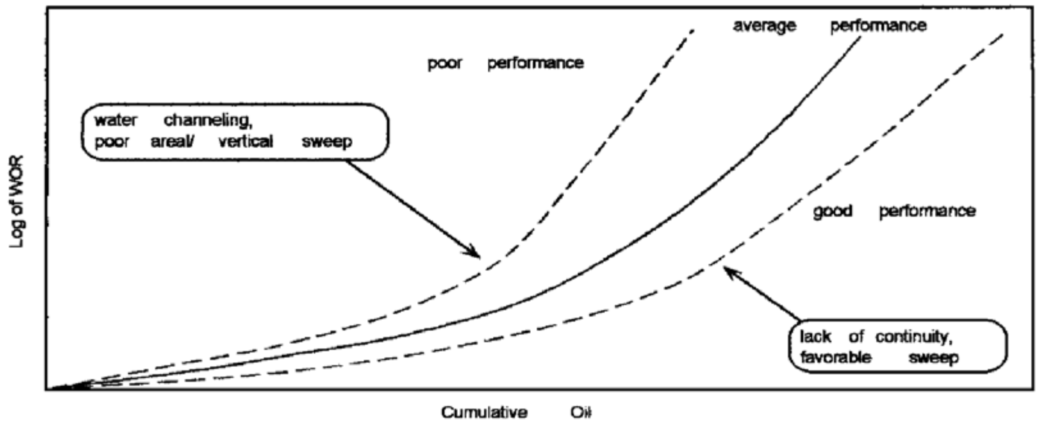


Figure 2. 2: Typical example of WOR plot for poor, average and good waterflood performances[13].

2.1.3 The X-Plot

Extrapolation of past performance on the "cut-cum" plot is a complicated task. The difficulty arises mainly because curve fitting by simple polynomial approximation does not result in satisfactory answers, in most cases.[11]

Ershaghi and Omorigie (1978) developed the X-Plot water flood analysis based on the semi-log linear relative permeability ratio (Equation 2.1) for intermediate saturation values, as follows:[14]

The fractional flow equation (after neglecting the capillary pressure and gravity terms) can be written as [11];

$$f_w = \frac{1}{1 + \frac{k_o}{k_w} \times \frac{\mu_w}{\mu_o}} \quad \text{(Equation 2.3)}$$

By introducing Equation 2.1 into Equation 2.3, it can be written as[11]:

$$f_w = \frac{1}{1 + Ae^{bS_w}} \quad \text{(Equation 2.4)}$$

According to Welge [15], the water saturation at the producing end could be expressed as [11]:

$$S_w = S_{av} - \frac{1 - f_w}{f_w^{'}} \quad \text{(Equation 2.5)}$$

where S_{av} is the average water saturation, and $f_w^{'}$ is the derivative of f_w with respect to S_w. Since [11]:

$$S_{av} = E_R(1 - S_{wc}) + S_{wc} \quad \text{(Equation 2.6)}$$

then:

$$S_w = E_R(1 - S_{wc}) + S_{wc} - \frac{1 - f_w}{f_w^{'}} \quad \text{(Equation 2.7)}$$

In these equations, E_R is the reservoir oil recovery, which is the volume of the oil recovered divided by oil in place [11].

The first derivative of Equation 2.4 will be [11]:

$$f_w^{'} = \frac{-Abe^{bS_w}}{(1 + Ae^{bS_w})^2} = -bf_w(1 - f_w) \quad \text{(Equation 2.8)}$$

On substitution, in Equation 2.5 will be [11]:

$$S_w = E_R(1 - S_{wc}) + S_{wc} - \frac{1}{b.f_w} \quad \text{(Equation 2.9)}$$

Putting Equation 2.9 into Equation 2.4 and solving it based on E_R gives:[11]

$$E_R = \frac{1}{b(1 - S_{wc})}\left[\ln\left(\frac{1}{f_w} - 1\right) - \frac{1}{f_w}\right] - \frac{1}{1 - S_{wc}}\left(S_{wc} + \frac{1}{b}\ln A\right) \quad \text{(Equation 2.10)}$$

By defining [11]:

$$m = \frac{1}{b(1-S_{wc})} \qquad \text{(Equation 2.11)}$$

$$X = \left[\ln\left(\frac{1}{f_w} - 1\right) - \frac{1}{f_w} \right] \qquad \text{(Equation 2.12)}$$

$$n = -\frac{1}{1-S_{wc}}\left(S_{wc} + \frac{1}{b}\ln A\right) \qquad \text{(Equation 2.13)}$$

Equation 2.10 can be expressed as [11]:

$$E_R = mX + n \qquad \text{(Equation 2.14)}$$

Therefore, a graph of fractional recovery (E_R) versus X results in a straight line (Figure 2.3). The straight line may be extrapolated to any desired water cut to obtain the corresponding recovery in future. But since the objective is always to project a waterflood performance into the future, only water cuts higher than 50% should be used in the linear regression model. The slope m and intercept n can be obtained from production data, if the swept volume is known. Other than predicting waterflood performance by extrapolating the linearity between X and E_R , permeability ratio constants a and b can also be computed, using the following equations [16]:

$$a = \frac{\mu_o}{\mu_{bw}} e^{-b\left[n(1-S_{wc})+S_{wc}\right]} \qquad \text{(Equation 2.14)}$$

$$b = \frac{1}{m(1-S_{wc})} \qquad \text{(Equation 2.15)}$$

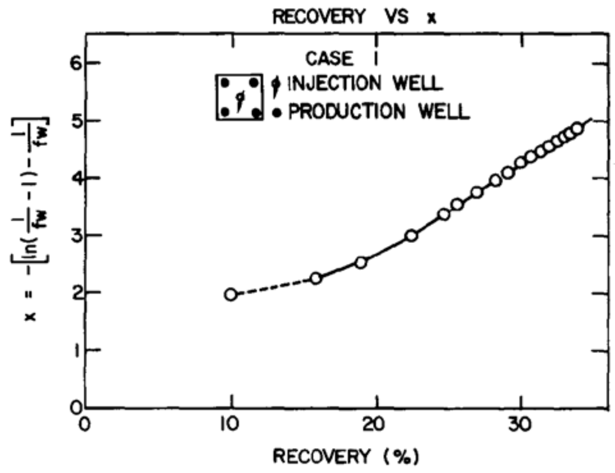


Figure 2. 3 An example of plot of X values versus Recovery obtained from Equation 2.2 [17].

2.1.4 The Hall Plot

A simple and efficient method of monitoring injection well performance at steady-state flow was proposed by Hall in 1963[18, 19]. Hall devised a qualitative approach to eliminate the complications of both pressure and injection rate variations[20]. This technique, used to analyze injection-well data, is based on a plot of cumulative pressure versus cumulative injection [21].

The Hall plot is a tool to analyze steady-state flow at an injection well. Originally, it was based on the radial flow P_i model [22]. According to this model [23]:

$$P_i = P_e + \frac{\mu}{2\pi kH} Ln \frac{r_e}{r_w} q_i \qquad \text{(Equation 2.16)}$$

where P_i and P_e are the downhole wellbore pressure (injection pressure) and reservoir pressure, respectively, q_i is the injection rate, μ is the injected fluid viscosity , k is formation permeability and H is the reservoir thickness. r_e is the well influence zone radius, which is the zone near the wellbore where the fluid pressure changes appreciably due to the injection. And r_w is the well-bore radius [23].

Equation 2.16 is based on several assumptions. The fluid is homogeneous and incompressible. The reservoir is vertically confined and uniform, both with respect to the permeability and the thickness. The reservoir is homogeneous, isotropic, and horizontal and gravity does not affect the flow. Consequently, the flow is radial. During the entire time of observations, the pressure at the distance equal to r_e is constant, and this distance is constant as well. In practice, not all, if any, of these assumptions are strictly satisfied. The injection interval usually covers multiple zones [23].

Equation 2.16 can be integrated with time to:

$$\int_{t_0}^{t}\left(P_i - P_e\right)d\tau = \int_{t_0}^{t}\frac{\mu}{2\pi k_w H}\ln\left(\frac{r_e}{r_w}\right)q_i d\tau$$ (Equation 2.17)

Then the plot of the integral:

$$\Pi(t) = \int_{t_0}^{t}\left(P_i - P_e\right)d\tau$$ (Equation 2.18)

versus the cumulative injection volume:

$$V(t) = \int_{t_0}^{t} q_i d\tau$$ (Equation 2.19)

at a constant ambient reservoir pressure, p_e , is a straight line whose slope is equal to $\frac{\mu}{2\pi k_w H}\ln\left(\frac{r_e}{r_w}\right)$, which is the reciprocal of the injectivity index, (*II*) [23] (Figure 2.3). Thus, a deviation from a straight line should signify alterations of the formation properties [19].

The Hall Plot, is an alternative to the transient well test analysis approach. Technically, it is very simple: just plot the time integral of the difference between the injection and reservoir pressures versus cumulative injection. The integration automatically filters out short-term fluctuations. The slope of lot is interpreted as an indicator of the average well injectivity. As inputs, the Hall method only requires the regular collection of injection rates and injection pressures that are a part of routine waterflood operations. At normal conditions, the plot is a straight line and kinks on the plot should indicate changes of injection conditions [23]. But it should be mentioned that if Hall s method is applied without a priori knowledge of the effective ambient reservoir pressure, then the conclusion that a kink in the plot is an indication of changes in the well injectivity can be wrong [23].

A typical Hall plot at different conditions is illustrated in Figure 2. Early in the life of an injection well the water-zone radius will increase with time, causing the slope to concave upward, as shown by Segment *ab* in Figure 2.3. After fill-up, Line *bA* indicates stable or normal injection. An increasing slope that is concave upward generally indicates a positive skin or poor water quality (Line *D*). Similar slopes may occur if a well treatment is designed to improve effective volumetric sweep. In this case, however, the slope will first increase and then stay constant. Line *B* indicates a decreasing slope, indicating negative skin or injection above parting pressure. The injection under the latter condition can be verified by running step-rate tests. A very low slope value, as shown by Line *bC*, is an indication of possible channelling or out-of-zone injection [2].

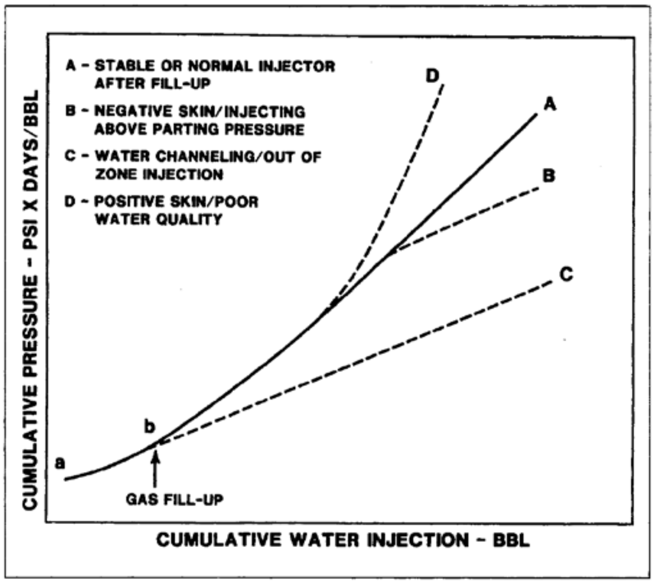


Figure 2. 4: Typical Hall plot for various injection well conditions[2].

2.2 Case study

In the next step in this chapter we will employ a small reservoir model to evaluate these monitoring techniques. In this analysis we are interested to see the efficiency of those methods in differentiating between normal and improved flooding performance. In other words we want to know:

1. What is the accuracy of these techniques in forecasting waterflood performance?

2. What kind of information can be obtained from each method?
3. Are these methods able to determine the optimum parameters in order to improve the waterflood performance?
4. What will be the advantages and disadvantages of each technique?

In order to reduce the complexity of the problem and focus more on the techniques rather than the challenges of complex models, a small reservoir model is used to carry out this analysis. The employed reservoir model is a standard example of the Reveal reservoir simulator that is mainly used to study water injection management. Two different injection schemes are defined for water injection.

In order to answer these questions all these methods are plotted for two injection schemes. Then for each method, the difference between the plots is analysed.

2.2.1 Model description

The model has 3 producers and 4 injectors that support them. The position of the wells has been shown in Figure 2.5. The model has 3 reservoir layers. The properties of the reservoir are given in Table 2.1.

Table 2. 1: The properties of the studied reservoir model.

Number of the cells	20×15×4
Initial oil in place	5.7522×10^{8} STB
Initial reservoir pressure	4000 psia
Temperature	100^{o} F
Compressibility	3×10^{6} 1/psi
Permeability	220-320 mD
Porosity (fraction)	0.2-0.25
Oil density	35 API
Bubble point pressure	2054 psia
Gas oil ratio	500 Scf/STB

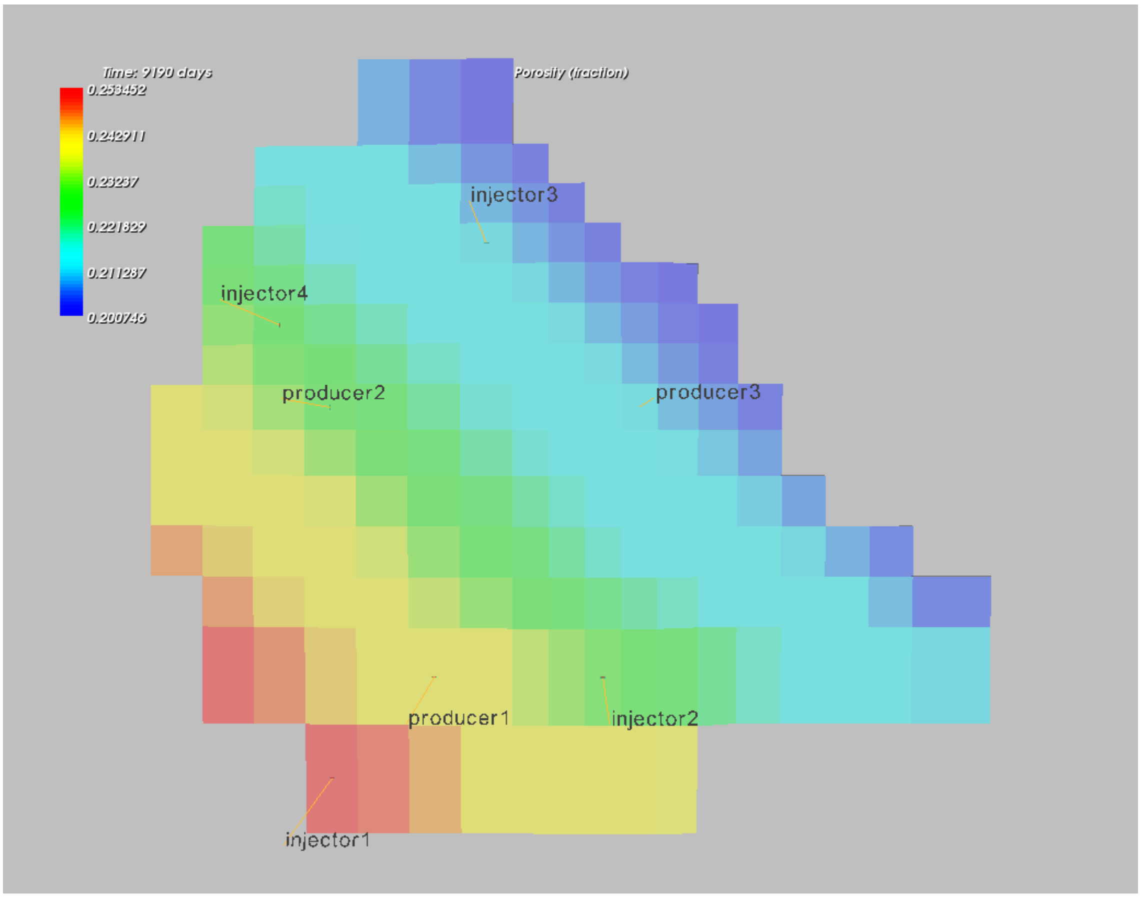


Figure 2. 5: Location of the wells in the reservoir (colours represent the reservoir porosity).

The model was set to produce for 40 years and the injection started from the beginning of production, for both injection scenarios. The same production scenario was employed for both injection schemes. The production wells were producing with constant surface liquid production, as shown in Table 2.2.

Table 2. 2: Allocated liquid production rate of each producer.

Production Wells	Liquid production (STB/day)
Producer 1	9000
Producer 2	9000
Producer 3	12000

For injection two scenarios were studied: the base case scenario and the water allocation management scenario (*WAM*).

2.2.1.1 Base case scenario

In this scenario the overall injection was controlled by voidage replacement by injecting with total voidage ratio (*VR*) of 1 for the whole 40 years of production. This means that we are injecting as much as we are producing from the reservoir. The total injection rate was equally distributed between injectors, so that, each injection well is injecting with 0.25 *VR*.

2.2.1.2 WAM scenario

In the *WAM* scheme, the total injection rate of the reservoir was based on voidage replacement with *VR* of 1, as in the base case scenario. For the first 20 years, the water distributed equally between the injectors. But for the next 20 years, at 5 year intervals, water was allocated to the injectors based on the inter-well connectivity between injectors and producers and also the performance of the production wells.

In order to do that, a capacitive resistive mode (*CRM*) [24, 25] is used to quantify inter-well connectivity between injectors and producers (the procedure briefly explained in Chapter 3 sections 3.2.3 and 3.3.3). Then the oil production index (*OPI*) is defined to describe the production well performance (briefly explained in Chapter 5 section 5.1.4). Then more water was allocated to the injectors that were highly connected to the producers with better *OPI*. This helped to improve the waterflood performance by promoting more oil recovery (Figure 2. 6) and less water production (Figure 2.7).

2.2.2 Plot Analysis

An Excel macro was developed in this research to plot all these performance plots, based on the results of the simulation imported from the reservoir model by the Reveal fluid flow simulator. This macro was able to plot all the above mentioned methods.

In the following section we will review the result the plot analysis for both injection schemes, for each surveillance technique.

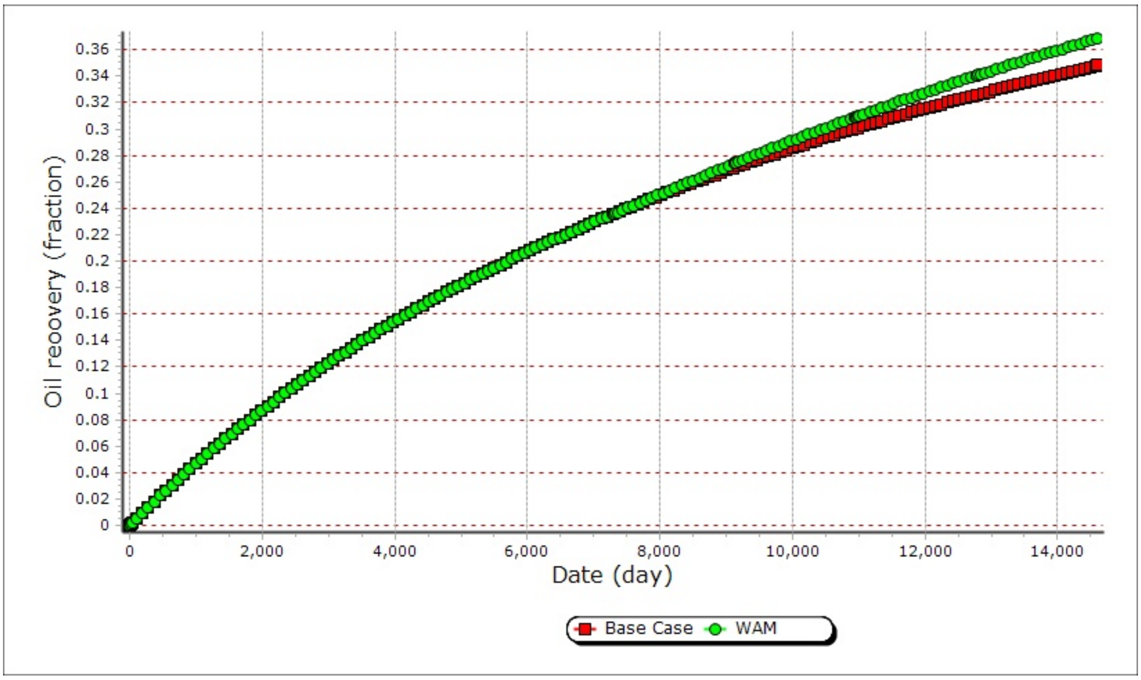


Figure 2. 6: Plot of oil recovery versus time for the base injection scenario and the WAM injection scenario.

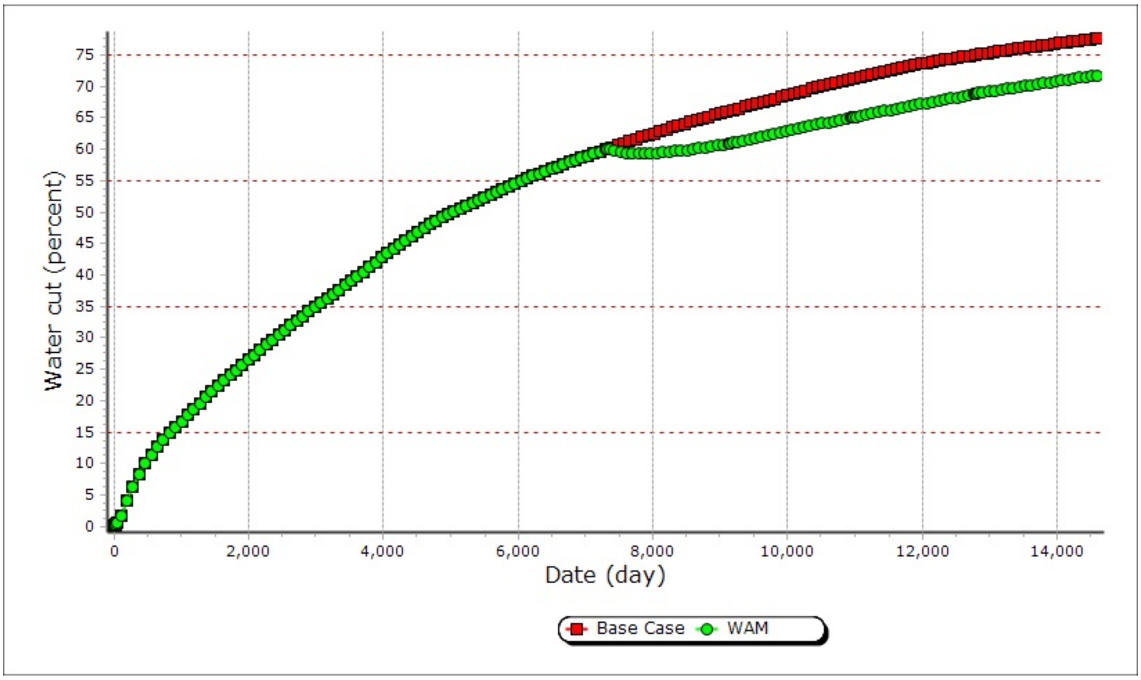


Figure 2. 7: Plot of water cut versus time for the base injection scenario and the WAM injection scenario.

2.2.2.1 *Cut-Cum plot*

Oil cut versus cumulative oil production was plotted on a semi-log scale, as shown in Figures 2.8 and 2.9.

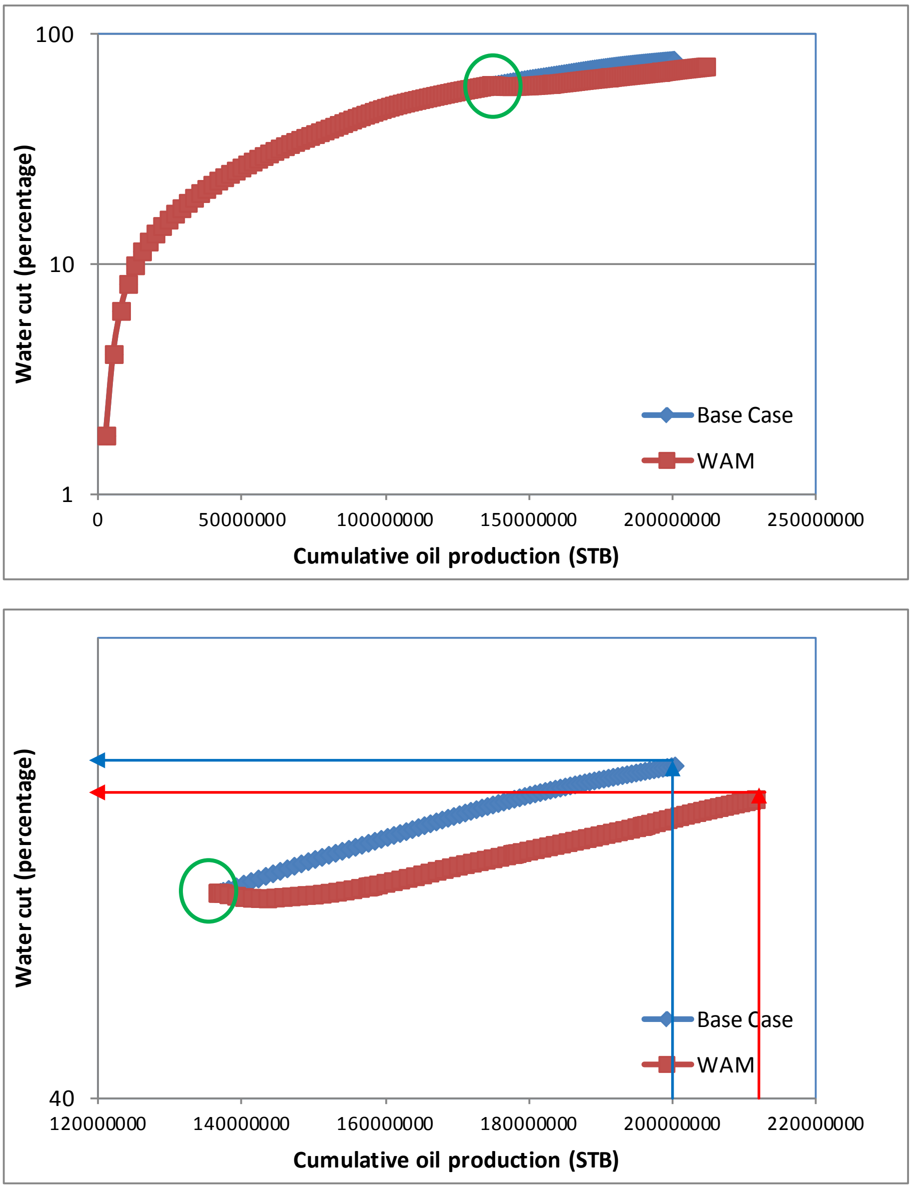


Figure 2. 8: Semi log plot of water cut versus cumulative oil production plot for base injection scenario and the WAM injection scenario.

As can be seen, there is a change in the trend of the *WAM* plot (see green circles in Figure 2.8) while this trend is constant for the base case. This is the time when water allocation management has been started. At the end of each plot it can be seen that more cumulative oil is produced in the lower water cut in the *WAM* scheme (red and blue arrows in Figure 2.8), which is a signature of improvement in the water injection efficiency. By fitting the cut-cum curve with proper polynomial, the future performance of waterflood in terms of future water cut or future cumulative oil production can be estimated from the cut-cum plot. Three types of polynomial are used to fit the cut-cum plot: linear, second order and third order polynomials. The results of extrapolation with these three polynomials were compared with the result of simulation, to evaluate the accuracy of this method. Table 2.3 shows the results of this analysis for the base case scenario after 10 years.

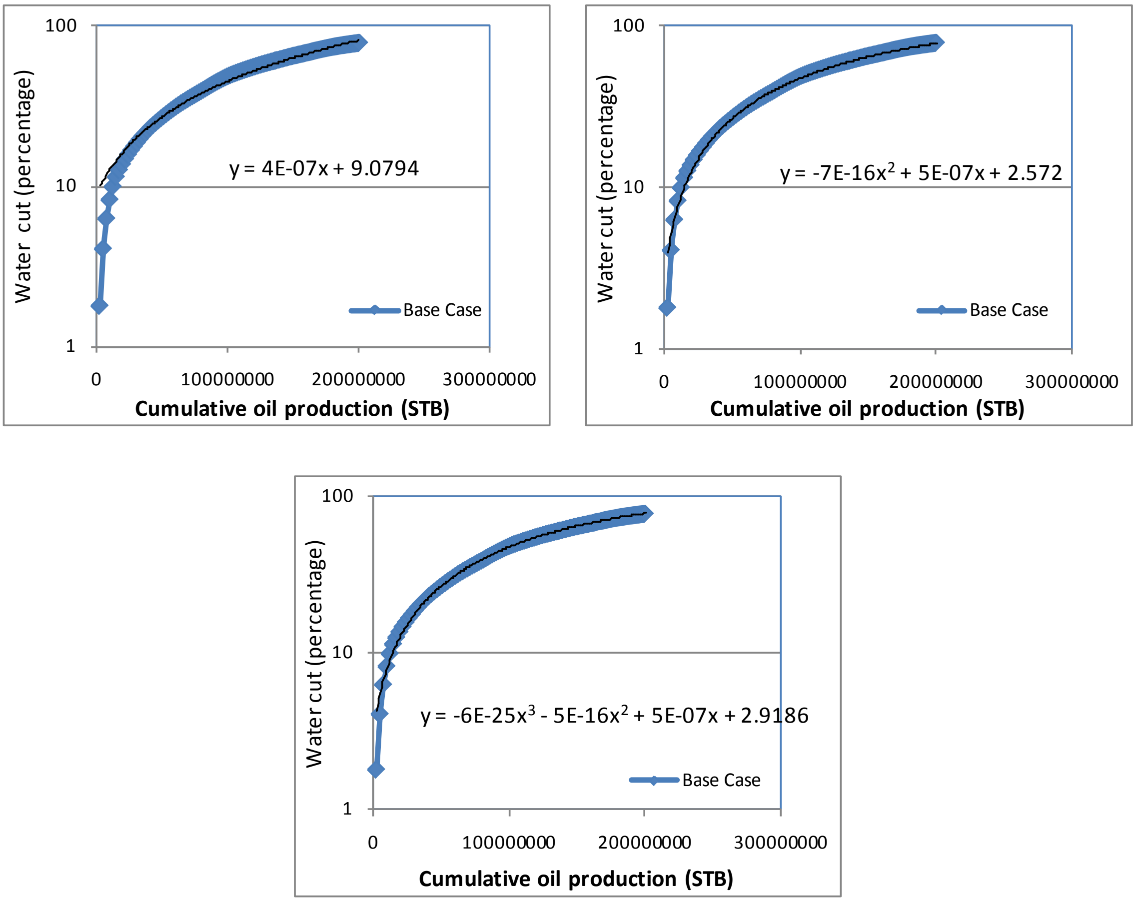


Figure 2. 9: Extrapolation of cut-cum plot for estimating future performance by employing first, second and third order polynomials.

Table 2. 3: Results of estimated future water cut for base case scenario from cut-cum plot extrapolation for cumulative oil production of 2.23×10^8 STB.

	Polynomials			
	linear	2^{nd} order	3^{rd} order	Simulation result
Estimated Water cut	98.1	79.1	89.4	81.3
Estimation error	20.5	2.7	9.8	---

The second order polynomial gave the best estimation compared to the simulation forecast. Looking at the results of other polynomials shows that fitting the cut-cum plot with a suitable polynomial has a significant effect on the result of extrapolation. We also tried to fit the polynomial to the last portion of the plot, for the water cut values greater than 70% (Figure 2.10). Using the second order polynomial gave a water cut of 79.5325%. This is not very different from the extrapolation resulted from all data points (estimation error of 2.25 %).

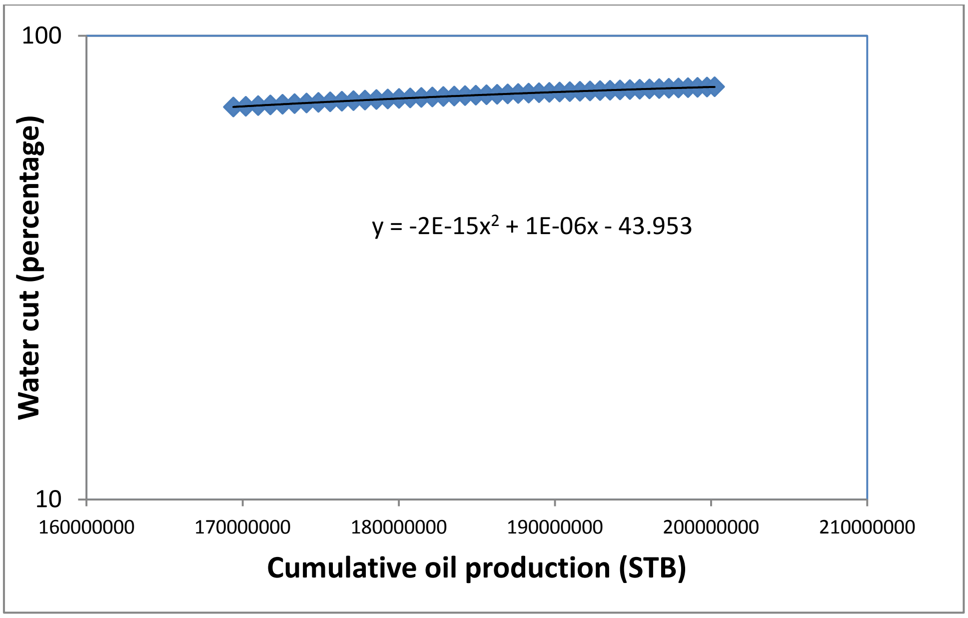


Figure 2. 10: Second order polynomial fitted for water cut values higher that 70%.

The second procedure was applied for extrapolation of the cut-cum plot for the *WAM* injection plan but only the 2^{nd} order polynomial was used to fit the curve. At first, the polynomial was fitted to all data points. Comparing the results of extrapolation with the simulation results shows an error of estimation of 8.14%, which is higher than the results of base case plot (Figure 2.11). This is because of the change in the curve after starting water allocation management. In order to obtain a more accurate extrapolation, the curve should be fitted after the change in the injection plan. Doing this reduced the error to 1.46% (Figure 2.12 and Table 2.4). So it is important not only to use a suitable polynomial, but also to select the correct portion of the plot to perform the extrapolation in the cut-cum plot.

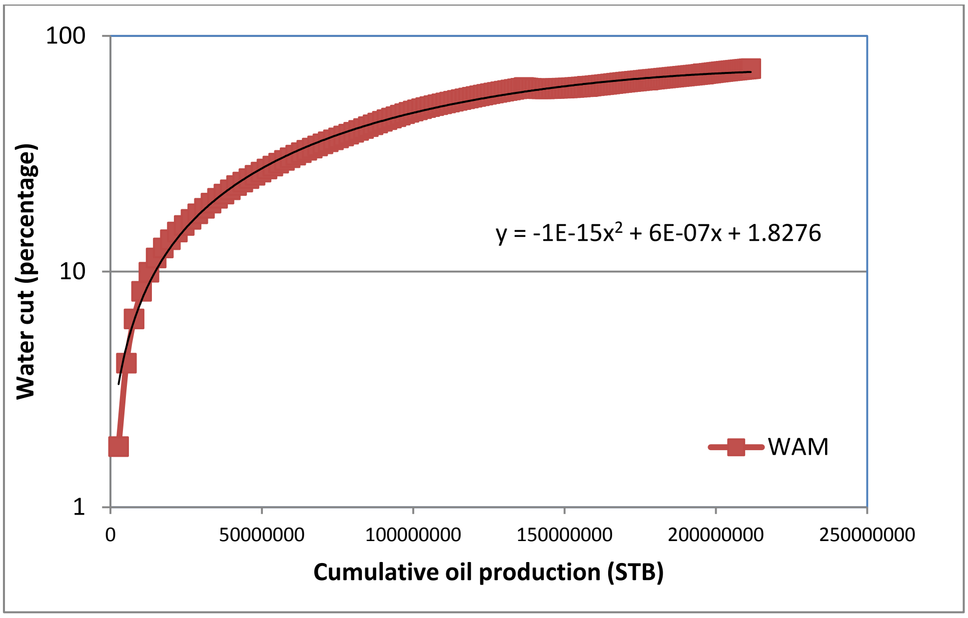


Figure 2. 11: Curve fitting of cut-cum plot for the WAM scenario when all data points are used.

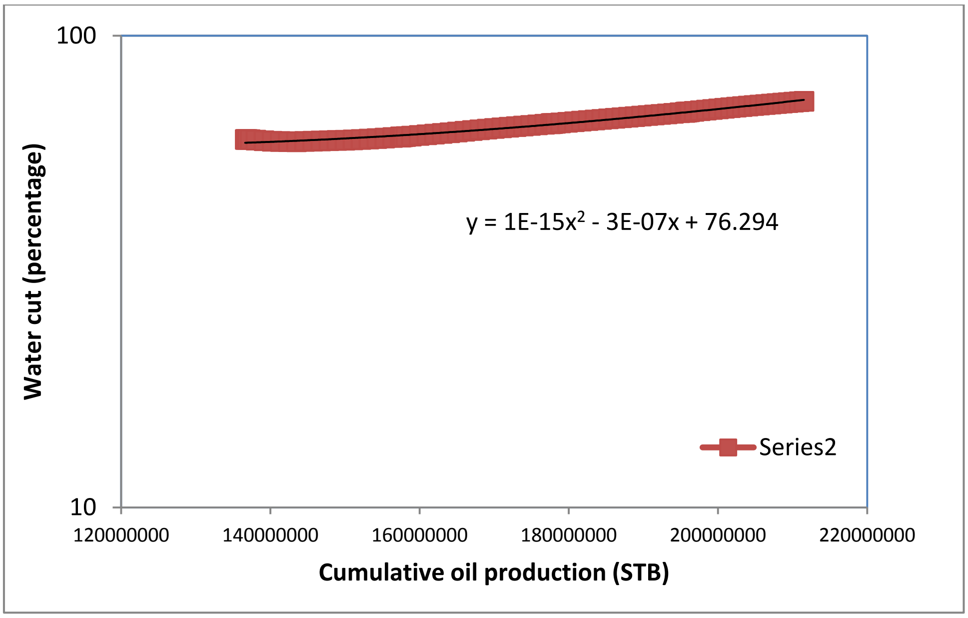


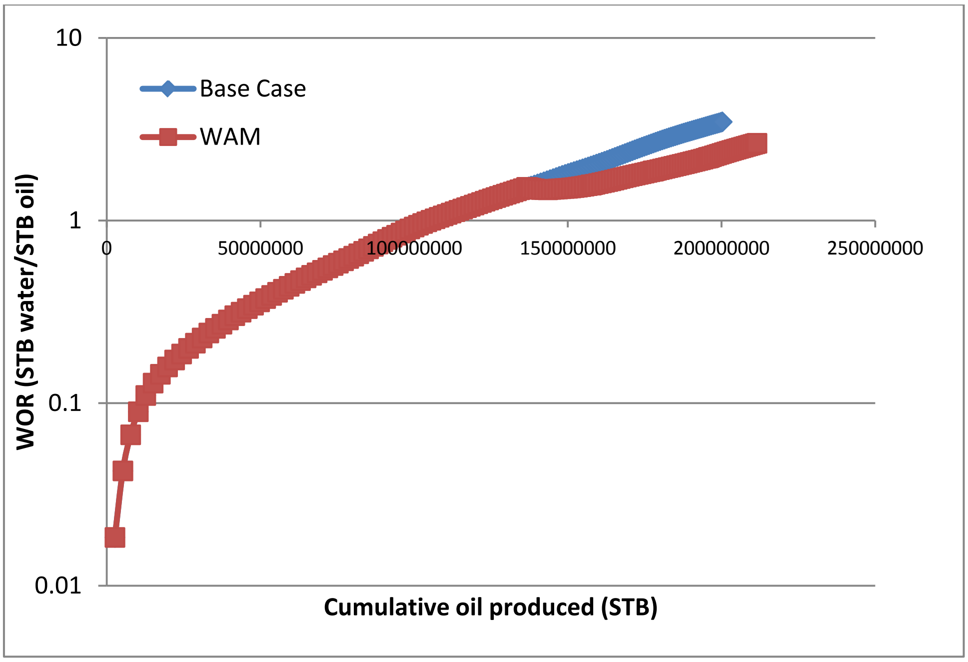
Figure 2. 12: Curve fitting of cut-cum plot for WAM scenario for data points after start of injection management.

Table 2. 4: Results of estimated future water cut for WAM injection scenario extrapolated to cumulative oil production of 2.38×10^8 STB.

	All data points	After WAM	Simulation
Estimated Water cut (%)	87.9	61.5	72.6
Estimation error (%)	21.1	15.2	---

2.2.2.2 WOR plot

Figure 2.13 shows the plot of *WOR* in logarithmic scale versus cumulative oil production for both water injection scenarios.



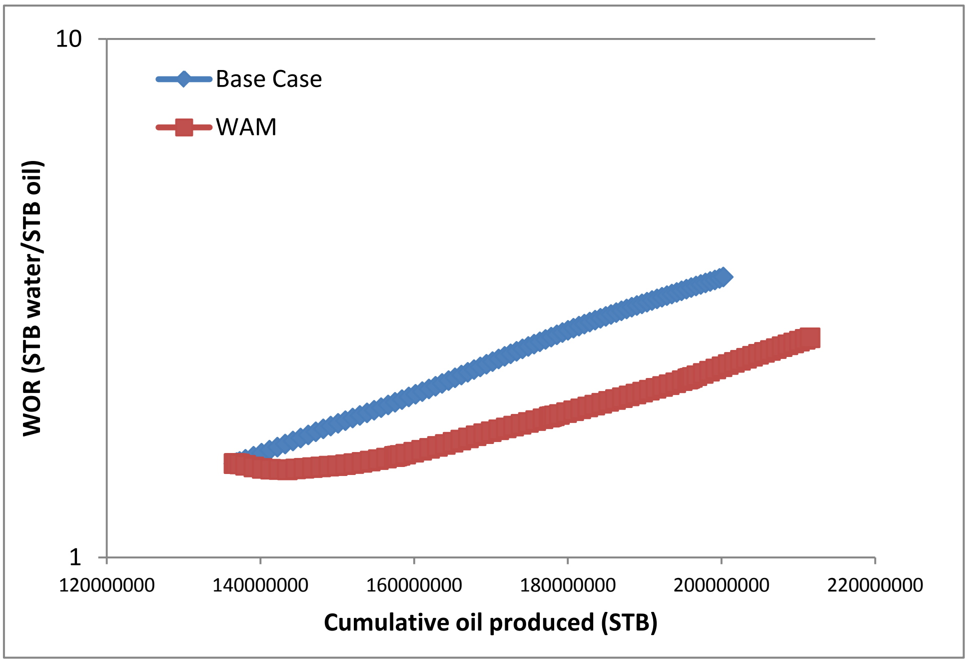


Figure 2. 13: WOR plots for base case and WAM injection schemes.

The *WOR* plot of the base case is a straight line, except at the beginning of production, but in the case of *WAM* there is a change in the slope of the plot, which represents the change in the injection plan. Compared with the cut-cum plot, the difference between the base case and the *WAM* plot is visually clearer. We can also observe that in the *WAM* plot, more cumulative oil production is obtained with less *WOR*.

According to Equation 2.2 there is a linear relationship between the logarithm of *WOR* and cumulative oil production, so a linear polynomial is employed to fit the curves. Curve fitting is therefore simpler, compared to cut-cum plots.

Results of extrapolation for the base case scenario are shown in Figure 2.14 and Table 2.5. The extrapolation has been done for whole set of data points and also for *WOR* values greater than 1.

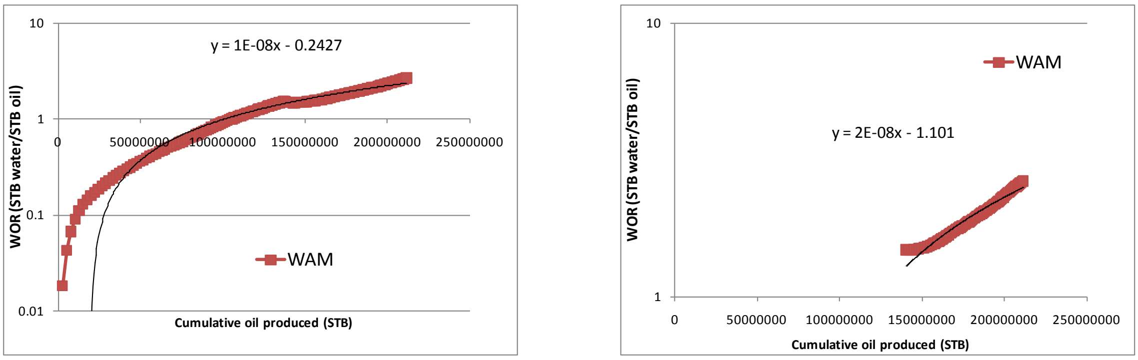


Figure 2. 14: WOR plot curve fitting for base case scenario when all data points are used and when WOR values higher than of 1 are employed.

Table 2. 5: Estimated future WOR for cumulative oil production of 2.23×10^8 STB for base case, from WOR plot.

	All points	WOR>1	Simulation
Estimated WOR (fraction)	5.1	4.6	4.3
Estimation error (%)	16.2	6.1	---

Although we are limited to the use of a linear equation for the *WOR* plot curve fitting, it still depends upon the portion of the curve we choose to do curve fitting. It is better to use that part of the plot which becomes a straight line.

Curve fitting has also been done for the *WOR* plot of the *WAM* injection scheme.

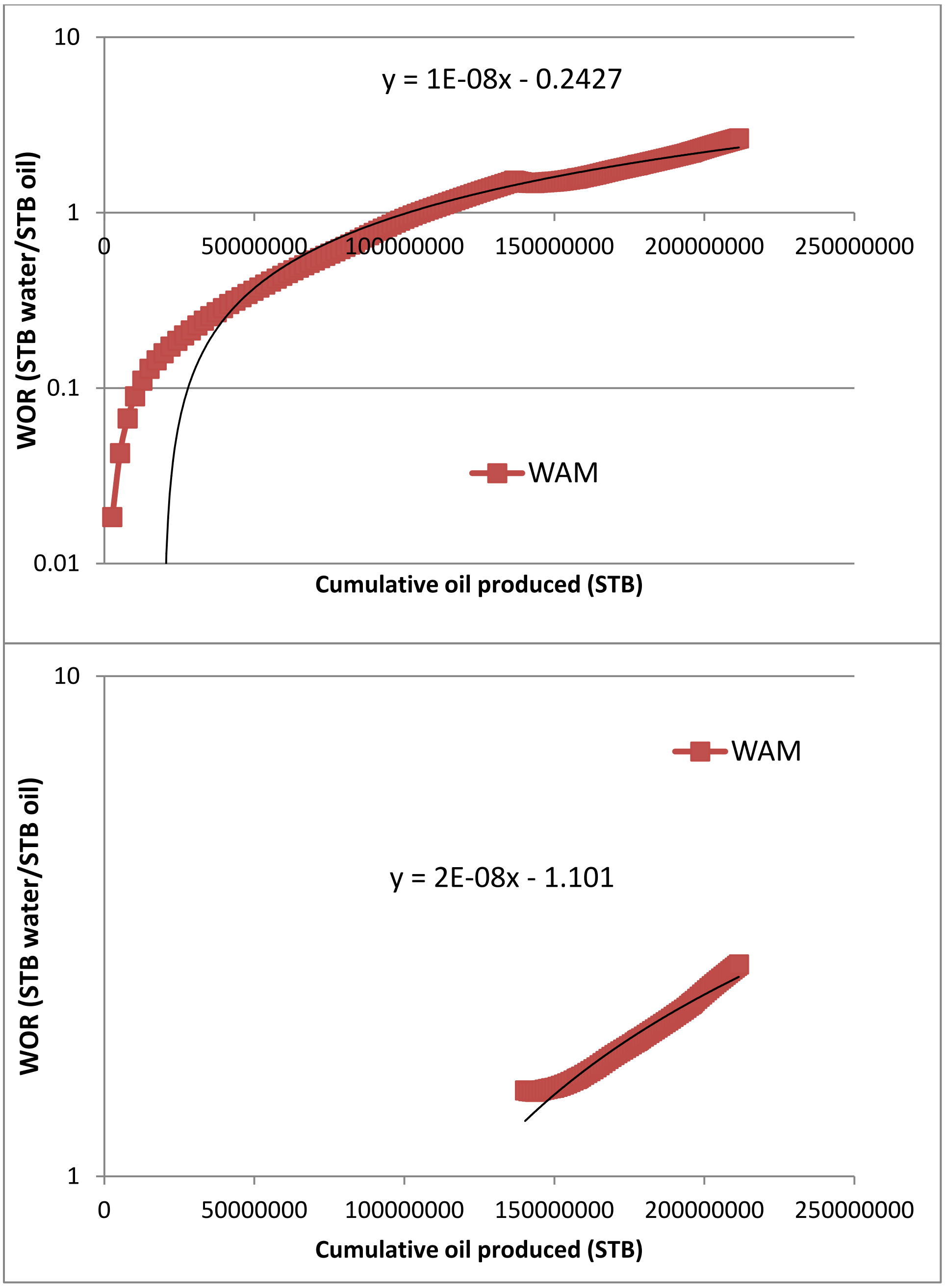


Figure 2. 15: WOR plot curve fitting for WAM injection scenario.

Table 2. 6: Estimated future WOR for cumulative oil production of 2.38×10^8 STB for WAM injection scheme.

	All points	After WAM	Simulation
Estimated WOR (fraction)	2.6	3.6	4.3
Estimation error (%)	39.9	16.1	---

Again, it can be concluded that the selection of data points is very important in curve fitting and extrapolation. In order to get an accurate forecast we have to consider that part of the curve which represents the last stable change in the water injection plan. One of the important factors that we need to consider for data selection is change in the operational conditions of the project, such as a change in the injection or production plan or changes in the number of active production and injection wells. As we saw when we used the data points after the start of water allocation management, the estimation become more accurate.

2.2.2.3 X-plot

The X-plot has been plotted for both waterflood projects in Figure 2.16. The green circle shows the change in the injection plan in the *WAM* scenario. After this point the curve shifts upward, showing improvement in the water flood project as more recovery is obtained. For example, as shown by the red and blue arrows for a specific *X* value (representing specific f_w), more oil recovery is achieved in the *WAM* plan.

Both plots have been extrapolated to obtain future reservoir recovery. Results of the X-plot curve fitting for both cases are illustrated in Figures 2.17 and 2.18.

Tables 2.7 and 2.8 represent the calculated future oil recovery from the extrapolation of X-plots for base case and *WAM* injection plans.

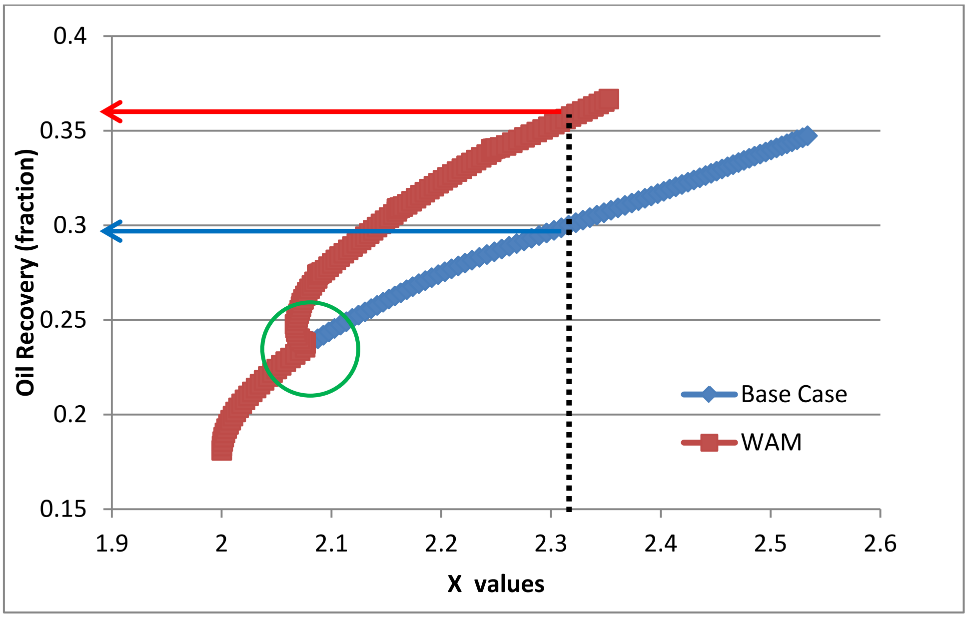


Figure 2. 16: Plot of oil recovery versus X values (X–plots) for base case and WAM injection schemes.

Looking to the extrapolation results of the X-plot shows that, although it has been recommended to use the plot for values of f_w more than 50% for recovery estimation, in this example, using the plot for values above 65 % provided us with more accurate estimation, especially in the case of *WAM*.

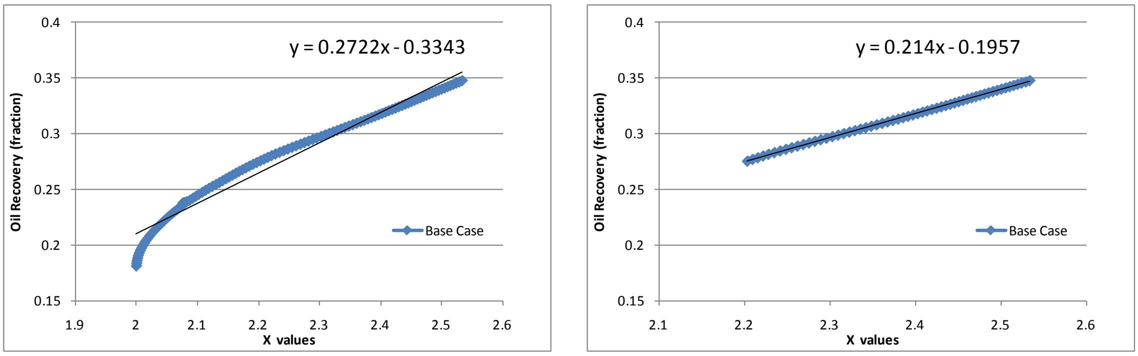


Figure 2. 17: X-plot curve fitting for base case scenario for all data points and for f_w higher than 65%.

Table 2. 7: Estimated future oil recovery from extrapolation results of X-plot at X value of 2.7 for base case.

	All data points	fw>65	Simulation
Oil recovery estimation (fraction)	0.40	0.38	0.38
Estimation error (%)	3.75	1.07	---

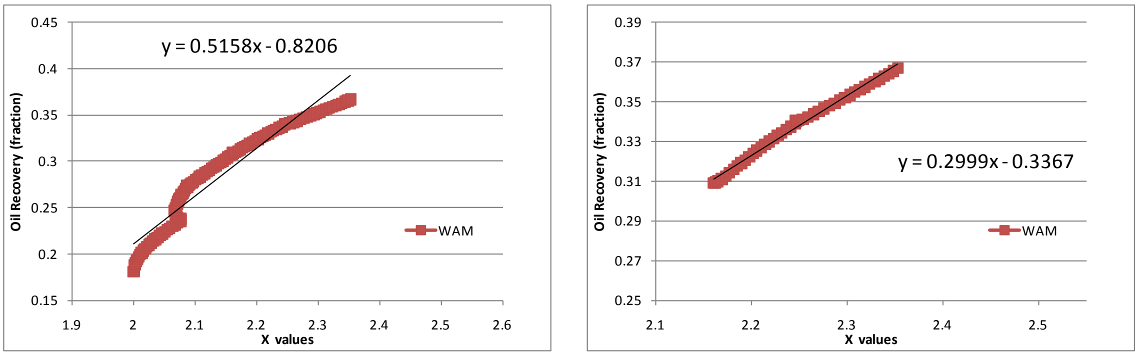


Figure 2. 18: X-plot curve fitting for WAM scenario for all data points and for f_w higher than 65%.

Table 2. 8: Estimated future oil recovery from extrapolation results of X-plot at X value of 2.7 for WAM scenario.

	All data points	f_w>65 (After WAM)	Simulation
Oil recovery estimation (fraction)	0.50	0.43	0.41
Estimation error (%)	22.38	5.06	---

Apart from the prediction of future oil recovery by X-plot, Equations 2.14 and 2.15 can be employed to determine the a and b constants of Equation 2.1. Table 2.9 shows calculated values of these constants for both water injection plans.

Table 2. 9: Determination of a and b constants.

	Base case	WAM	Real Values
m	0.21	0.29	---
n	0.19	0.33	---
a	3.60	4.22	4.75
b	5.19	3.70	3.92

The values of a and b can be used to produce an effective field relative permeability plot knowing the estimates of S_{wc} and the viscosity ratio.

2.2.2.4 Hall plot

Unlike the previous methods that applied for the whole reservoir, the Hall plot can be generated for each individual injector. The following Figures, 2.19 and 2.20, illustrate the Hall plots for all the injection wells in both injections schemes.

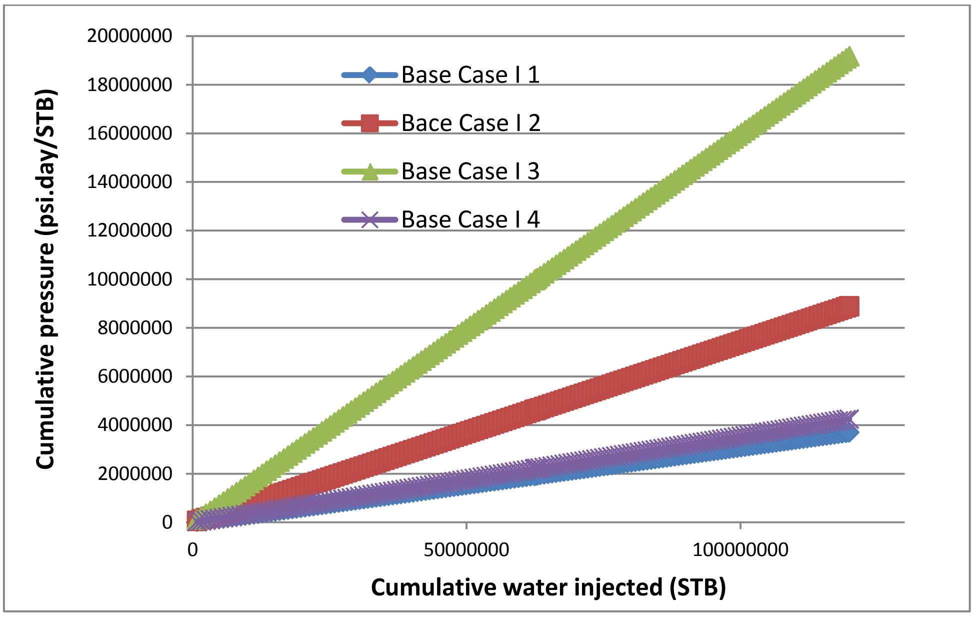


Figure 2. 19: Determined Hall plots of each injection well for base case injection scenario.

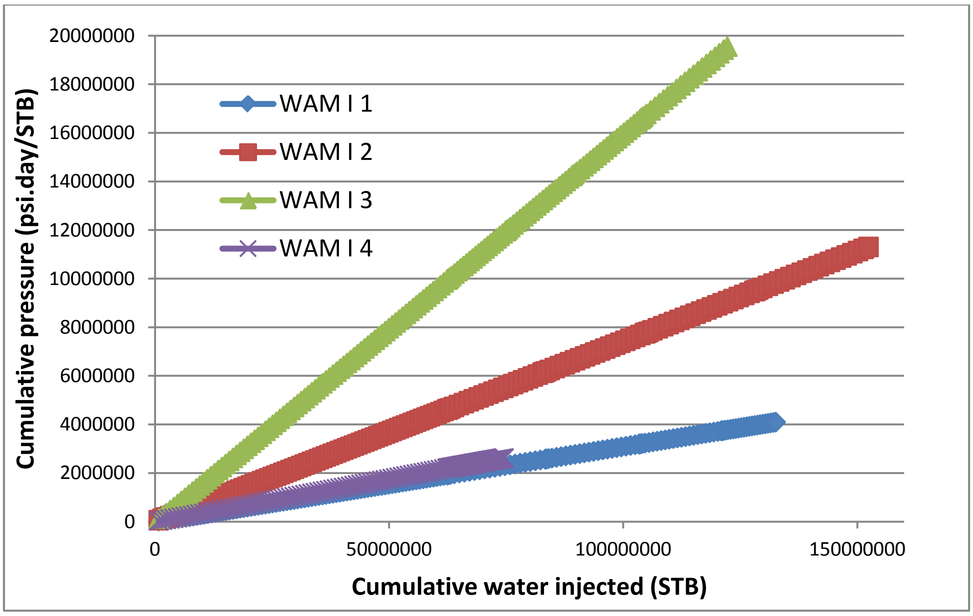


Figure 2. 20: Determined Hall plots of each injection well for WAM injection scenario.

Looking at the base case injection well Hall plots shows that the same amount of water was injected from each injector but different cumulative pressure was obtained. The slope of the Hall plot is the reciprocal of the injectivity index of the injection wells. Those wells with more cumulative pressure have steeper slopes, showing these wells have lower injectivity indexes. Since *VR* is 1, the reservoir pressure is almost constant, therefore to inject the same amount of water, injection pressures need to be increased, resulting in higher cumulative pressure for these wells (injectors 3 and 2).

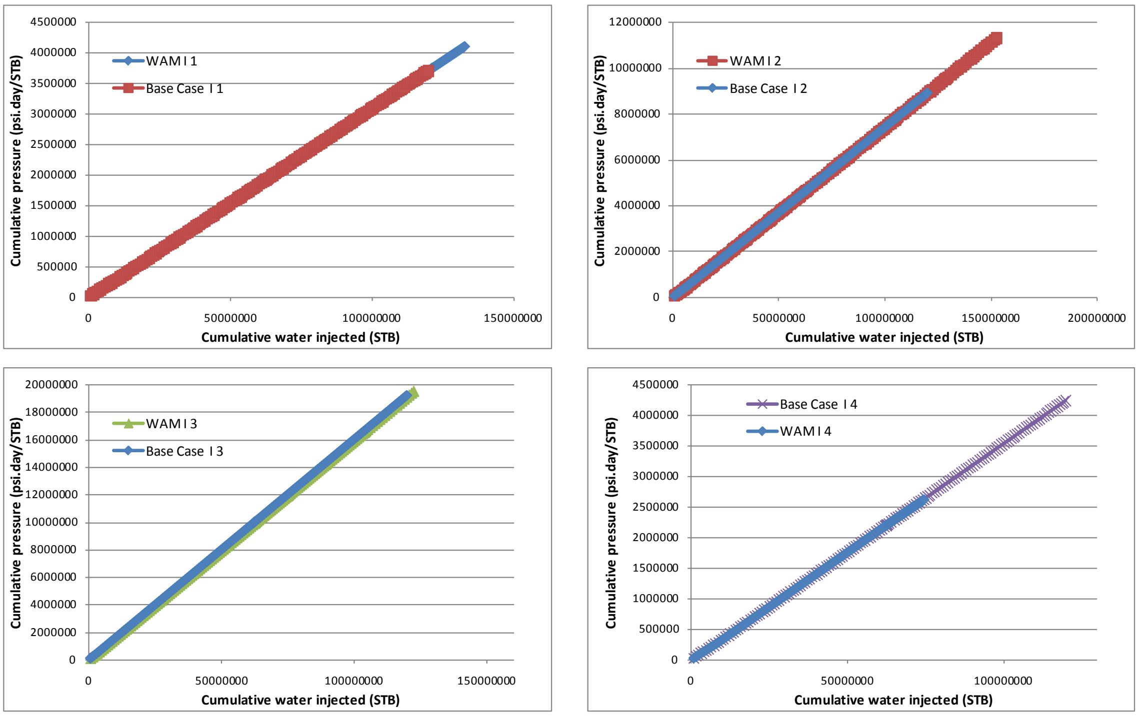


Figure 2. 21: Comparison of the Hall plot of each injector in base case and WAM injection scenarios.

Comparing the Hall plot of each injector in both injection plans (Figures 2.21) shows that the amount of water that injected from each injector has changed from the base case injection scenario to the *WAM* injection scheme and therefore, different cumulative pressures are obtained from each of them. This is the only difference that we can find between these two injection plans from the Hall plot.

The slope of the Hall plots for each injection well in both scenarios remains constant. Therefore, we can conclude that there is no sign of positive or negative skin or change in injectivity index.

The injectivity index of each injection well has been calculated from the slope of their Hall plots (Table 2.10). Since the input data came from reservoir simulation and there was no change in the injectivity index of the wells, the results are in very good agreement with the average injectivity index of the wells obtained from simulation.

Table 2. 10: Calculated injectivity index from Hall plots for each injection well.

	I 1	I 2	I 3	I 4
Slope	0.03	0.07	0.15	0.03
II from Hall plot	32.36	13.51	6.25	28.40
II from simulation	32.38	13.48	6.25	28.65

2.3 Reservoir voidage management

This section aims to develop a new voidage replacement plan for the reservoir in order to delay the breakthrough time as much as possible, maintain the reservoir pressure, and decrease water production. Again, in order to reduce the complexity of the problem and focus more on the techniques rather than the challenges of complex models a simple horizontal reservoir is employed in this analysis. The reservoir produces with constant liquid production rate. A sensitivity analysis was carried out to monitor the effect of different injected volumes of water on pressure maintenance and oil recovery from the reservoir. The result of this sensitivity analysis was then used to define a suitable reservoir voidage strategy.

2.3.1 Model description

Figure 2.22 shows the one layer, homogenous and horizontal reservoir model which is used in this study. This model contains one production well and one injection well. The properties of this model are given in Table 2.11.

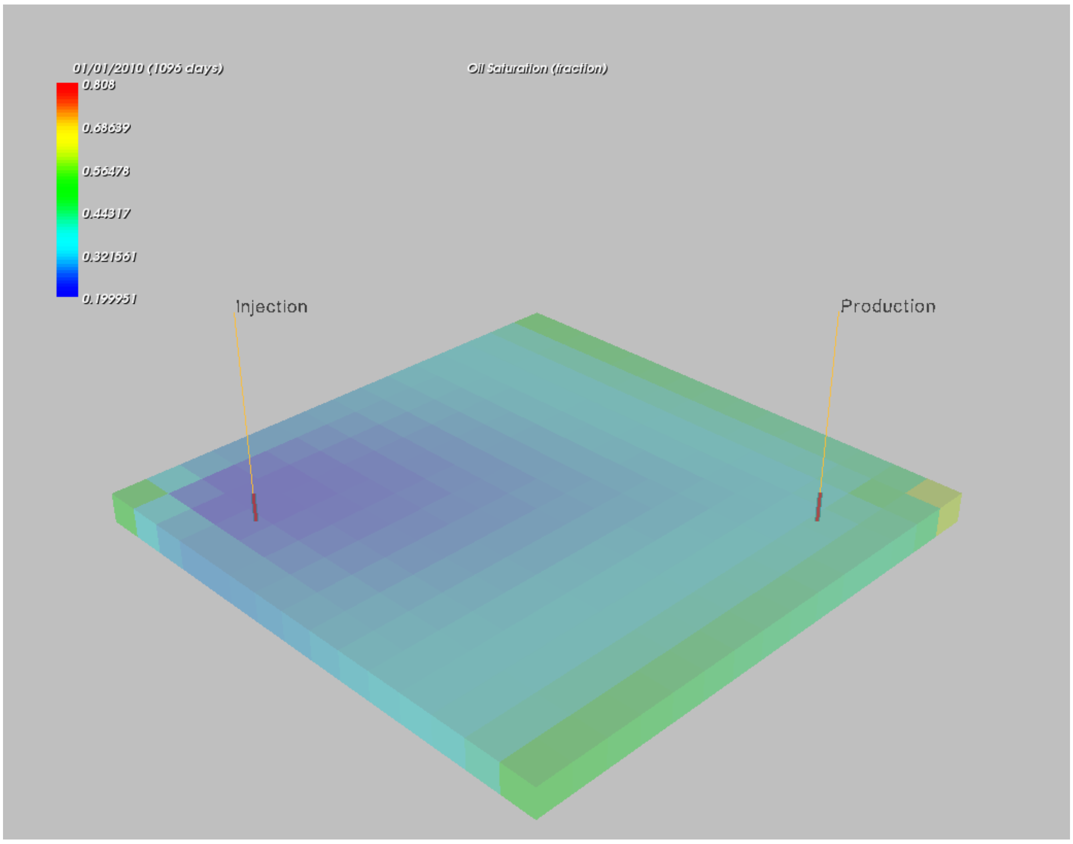


Figure 2. 22: Schematic of the reservoir model (colours representing oil saturation).

Table 2. 11: Properties of the studied reservoir model.

Dimension	Depth	Pressure	Temperature	Permeability	Porosity	Net to gross
450×450×25 ft3	5000 ft	5000 psig	200 ^{o}F	500 mD	30%	100%

2.3.2 *Methodology for reservoir voidage management*

This section describes the steps in setting up the model and defining the voidage replacement plan:

1. The reservoir was set to produce with constant liquid production of 250 STB/day for three years. The voidage replacement strategy was applied for water injection and water injected into the reservoir with constant voidage ratio (*VR*) from beginning of the oil production until the end of the production time. A sensitivity study was done to see the effect of different injection volumes of water on breakthrough time and final oil recovery. Figures 2.23 to 2.25 show the results of injection with different *VRs* from 0.1 to 1.0.

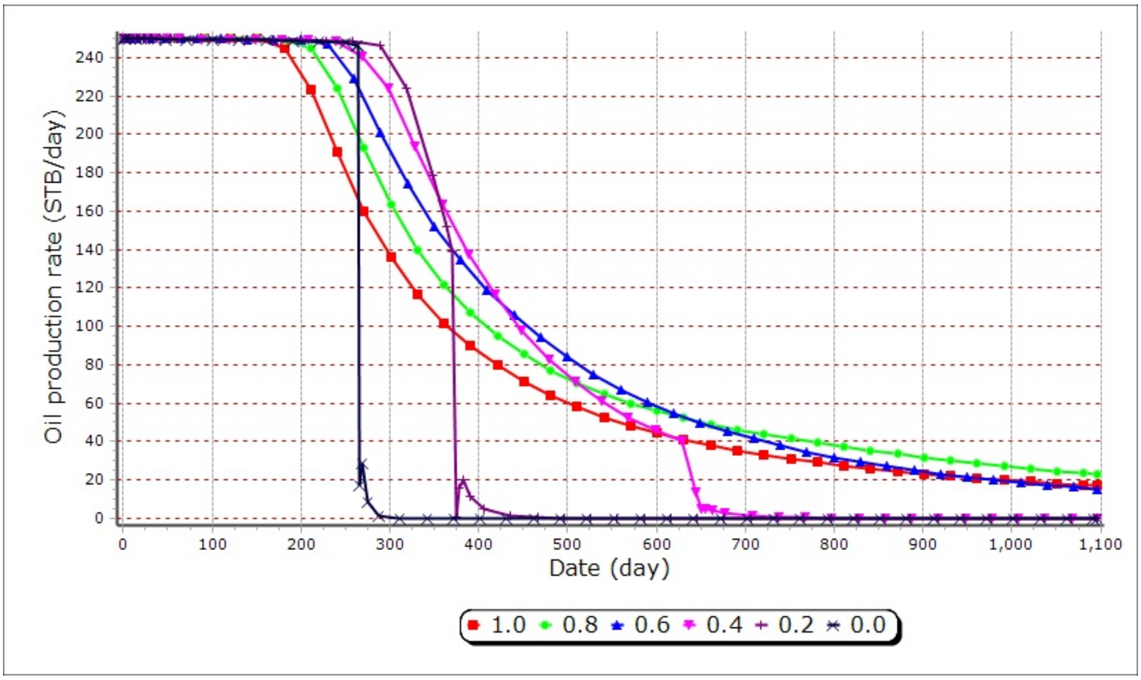


Figure 2. 23: Oil production rate versus the time for injection with different VRs.

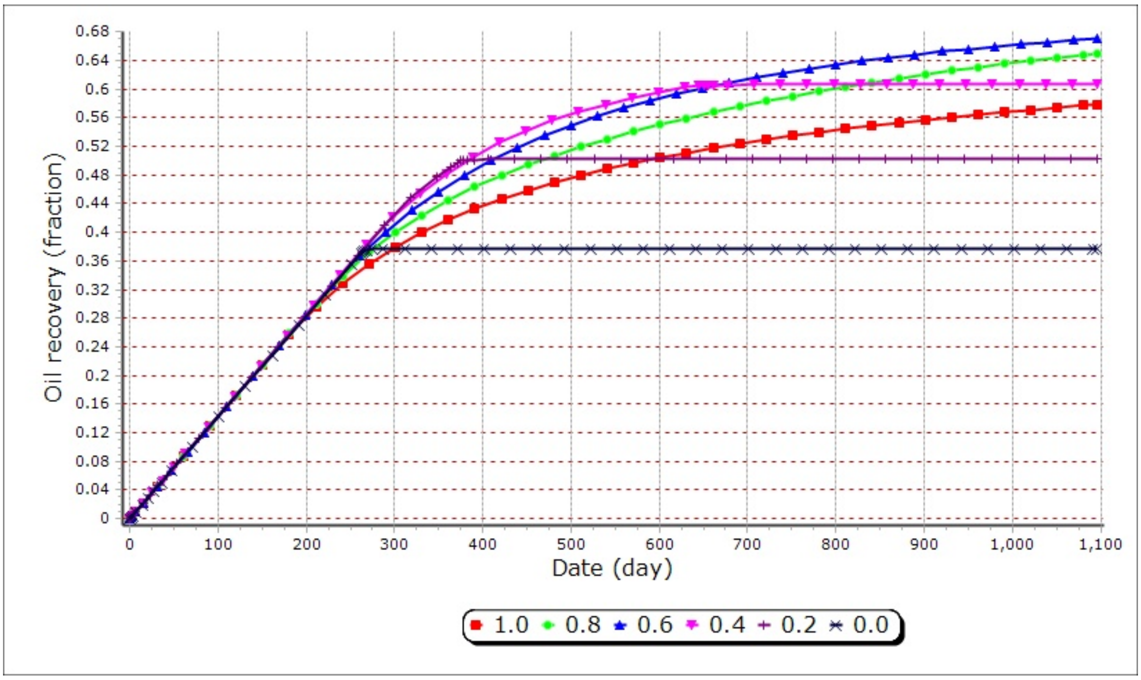


Figure 2. 24: Oil recovery versus time, obtained from injection with different VRs.

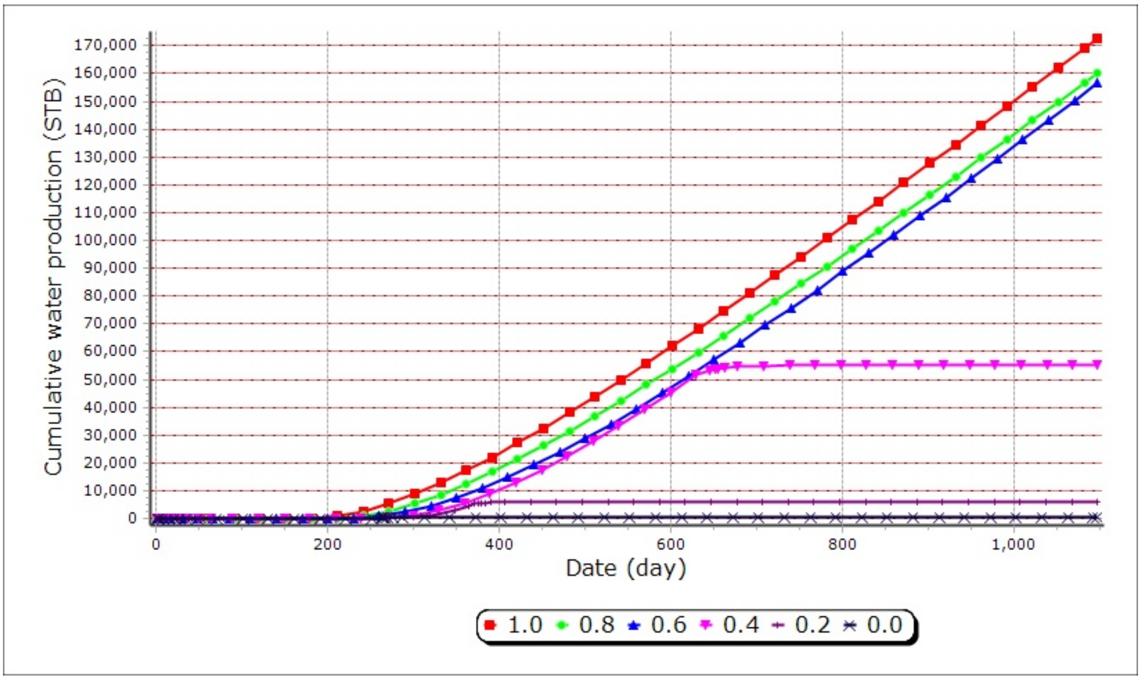


Figure 2. 25: Cumulative water production from injection with different VRs.

As expected, this analysis shows that:

a) More injection of water will cause early break through, less oil recovery with more water production but better pressure maintenance.

b) Less injection will delay breakthrough time and cause more oil to be swept but cannot maintain reservoir pressure and, as we can see, with *VRs* of less than 0.6, at some point, reservoir pressure (Figure 2. 25) is not enough to support production any more.

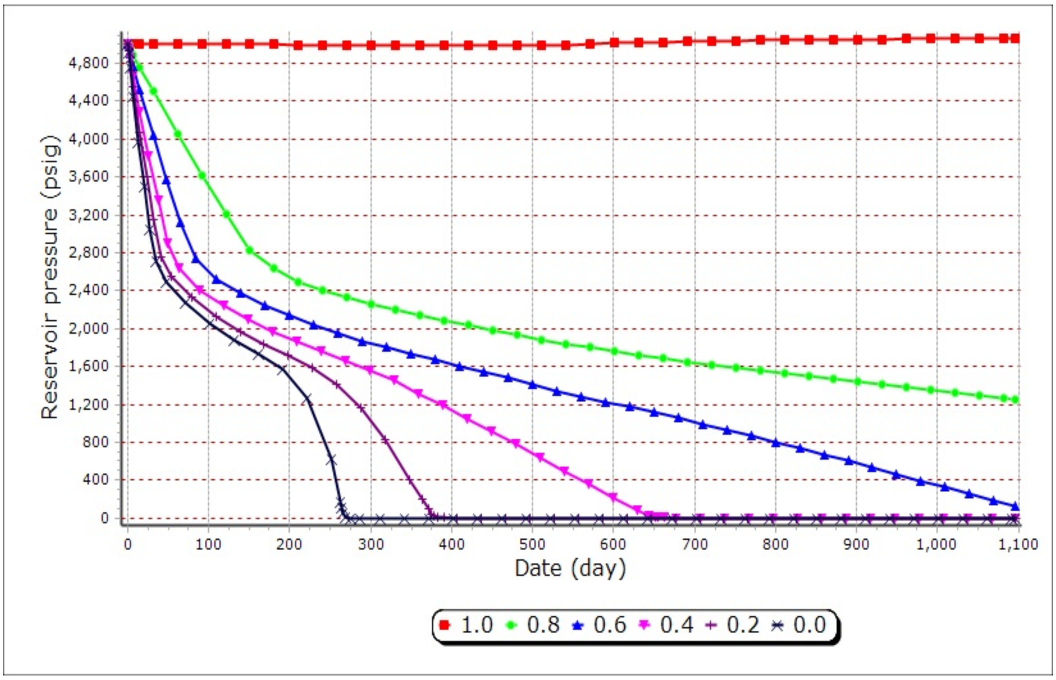


Figure 2. 26: Change in the reservoir pressure versus the time during injection with different VRs.

Even with a *VR* of 0.6, the reservoir pressure is not enough to deliver the produced fluid to the surface. We therefore need to define suitable reservoir pressure, in other word the minimum reservoir pressure required to produce from the reservoir at the planned production rate.

2. In order to define the suitable reservoir pressure, a well model is designed for the production well. This well is employed to run a sensitivity analysis on the reservoir pressure (Figure 2.27) by analysing the well model. According to this sensitivity study, the minimum reservoir pressure (P_{rmin}) in order to produce from the production well is 2400 psi. But since the bubble point pressure is 2630 psi and we do not want to produce below the bubble point pressure, 2750 psi selected as the minimum reservoir pressure.

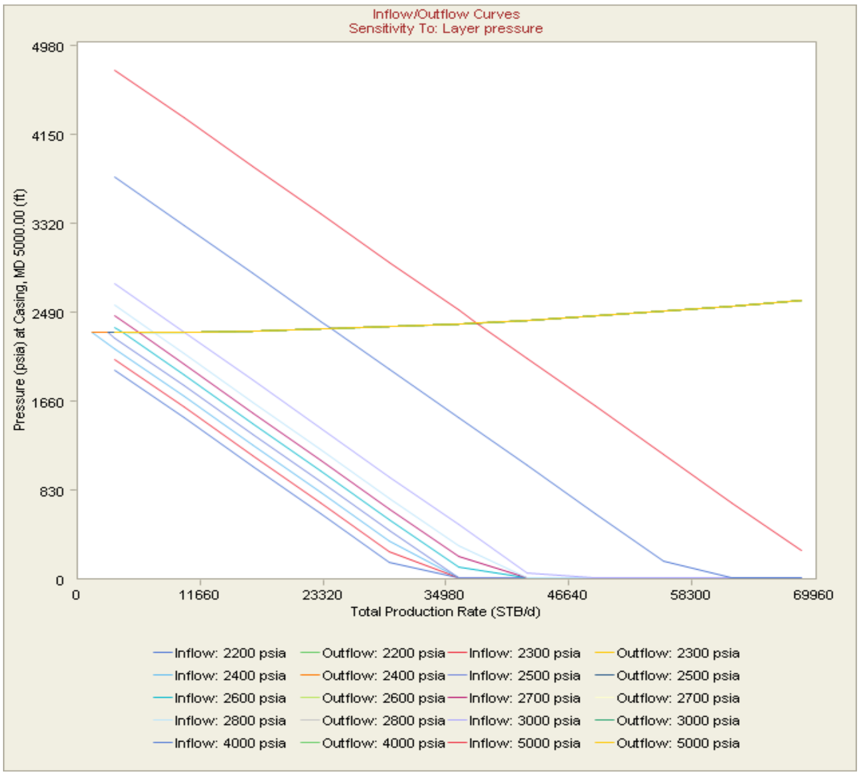


Figure 2. 27: Sensitivity analysis on the well outflow performance to determine the minimum required reservoir pressure.

3. After determining the minimum reservoir pressure, the following procedure is proposed to define the reservoir voidage replacement plan:
    1) The reservoir pressure is allowed to decline to decline the suitable reservoir pressure before the breakthrough time, by starting to inject with *VR* less than 1.
    2) As soon as the pressure falls to the minimum pressure, injection of VR 1 is commenced to maintain the pressure at the minimum level.
4. The starting *VR* for injection should be selected in a way to delay breakthrough time as long as possible. Sensitivity analysis on injection with different *VRs* shows the breakthrough will happen after injection of 45000 *STB* cumulative water volume (Q_{wibt}) into the reservoir. Therefore, based on this strategy it can be said:

$$Q_{wibt} = Q_{wibt1} + Q_{wibt2} \qquad \text{(Equation 2.20)}$$

in which Q_{wibt1} and Q_{wibt2} are the cumulative water injected by the first and second *VRs*, before breakthrough time. The first important parameter is the time of starting the injection with the second *VR* (which is 1 in this procedure). Let us call this time the time of starting pressure maintenance (t_{pm}). t_{pm} should be the time in which reservoir pressure declines to the minimum required reservoir pressure (P_{rmin}). According to the definition of compressibility we can say:

$$dV = -C \times V_{pv} \times dP \qquad \text{(Equation 2.21)}$$

In which dV is the depleted volume of the reservoir, C is the reservoir total compressibility, V_{pr} is the reservoir pore volume and dP is the change in the reservoir pressure. dV is the produced fluid from the reservoir minus the injected volume of the water into the reservoir:

$$dV = t \times B_o q_l (1 - VR) \qquad \text{(Equation 2.22)}$$

in which the t is the duration time of the production. Therefore t_{pm} can be estimated by the following equation:

$$t_{pm} = \frac{-C \times V_{pv} \times (P_r - P_{r\min})}{B_o \times q_l (1 - VR)} \qquad \text{(Equation 2.23)}$$

in which P_r is the initial reservoir pressure and q_l is the liquid production rate of the reservoir. Table 2.12 shows the estimated t_{pm} for different *VRs*.

Table 2. 12: Calculated t_{pm} for each injection volume of water, in terms of different *VRs*.

VR	t_{pm} (days)
0.9	330
0.8	180
0.7	120
0.6	85
0.5	70

5. The important question is what *VR* should be selected to start the injection. The governing parameter is the time of breakthrough. Since the voidage replacement strategy is used for injection, the injection rate is:

$$q_i = VR \times B_o q_l \qquad \text{(Equation 2.21)}$$

In which q_i and q_l are injection rate and liquid production rate respectively and B_o is the oil formation volume factor. According to Equation the 2.21, we can say:

$$Q_{wibt} = t_{pm} \times VR_1 \times q_l + t_2 \times VR_2 \times q_l \qquad \text{(Equation 2.22)}$$

in which t_2 is the duration time of injection with the second *VR* (which is 1 in this case) and it will be:

$$t_2 = \frac{Q_{wibt} - t_{pm} \times VR_1 \times q_l}{q_l} \qquad \text{(Equation 2.23)}$$

and the breakthrough time will be:

$$t_{bt} = t_{pm} + t_2 \qquad \text{(Equation 2.24)}$$

Figure 2.28 shows the calculated breakthrough time for each combination of *VRs*.

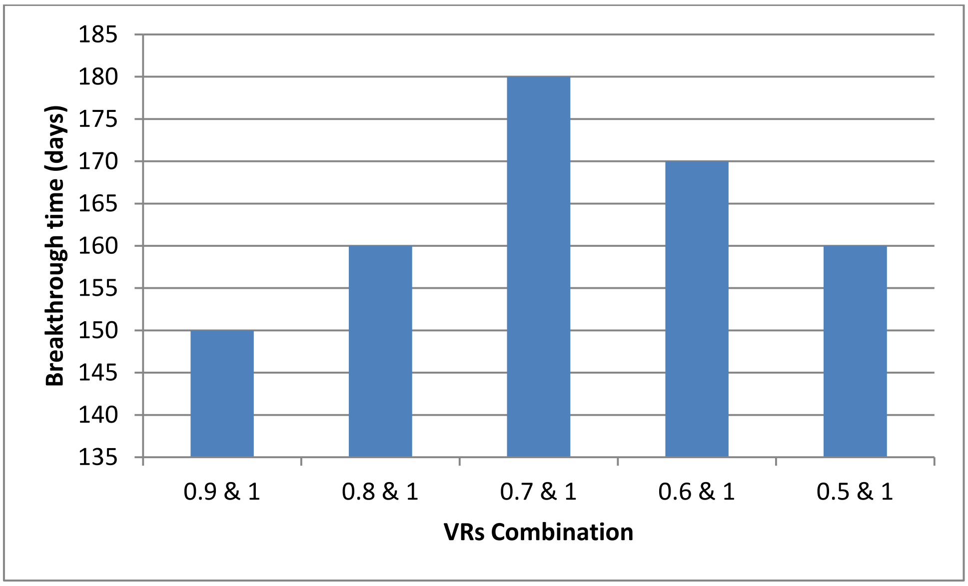


Figure 2. 28: Calculated breakthrough time for different starting injection VRs.

As can be seen, the maximum breakthrough time obtained is when we start to inject with a *VR* of 0.7.

Therefore, according to the proposed methodology, the voidage replacement plan for this reservoir will be to start to inject with *VR* of 0.7 for 120 days then increase the *VR* to 1 for the rest of the production.

The following figures compare the results of new pressure maintenance strategy with the base case plan in which water was injected with *VR* of 1.

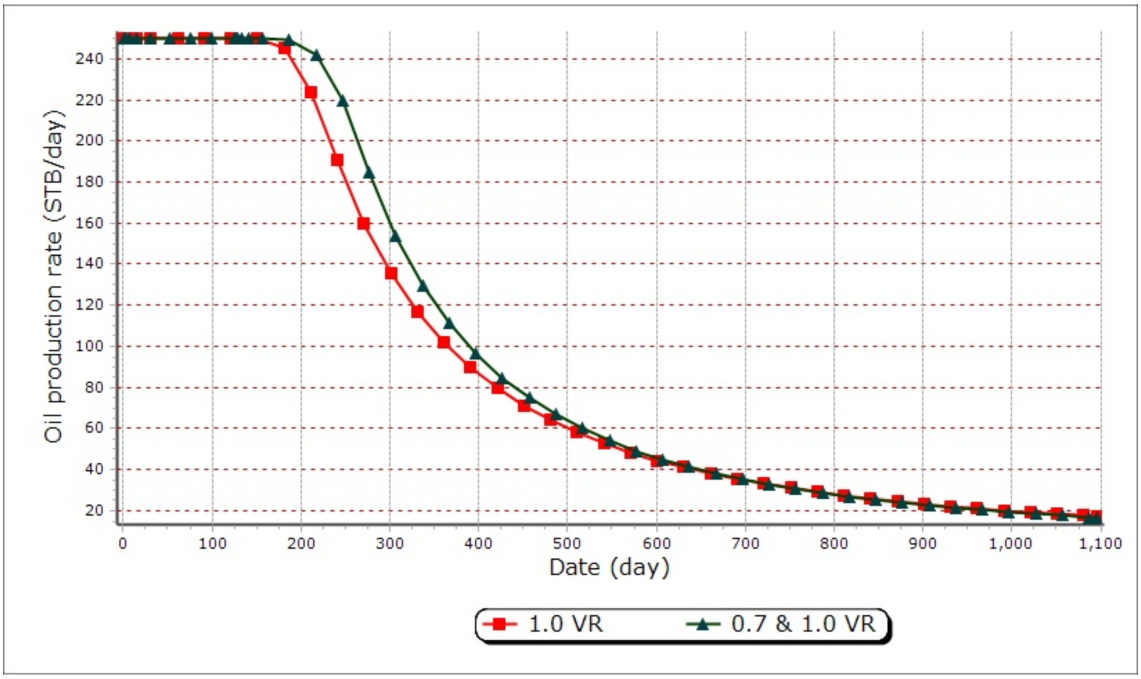


Figure 2. 29: Oil production rate versus time for base case and voidage management injection scenarios.

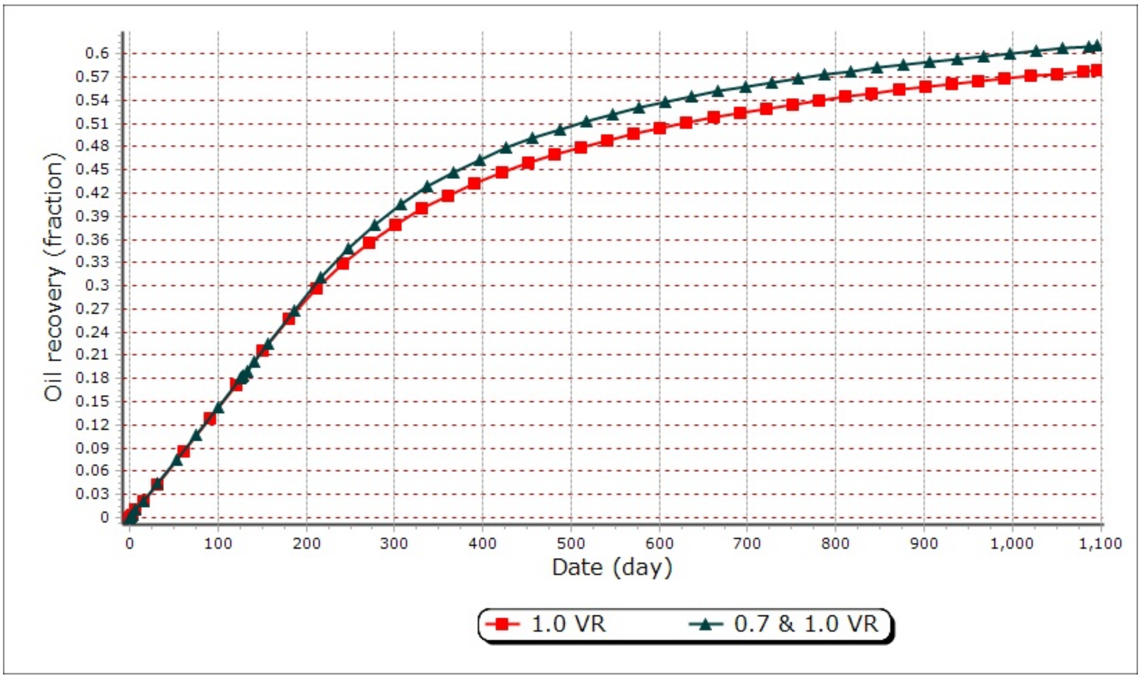


Figure 2. 30: Plot of oil recovery versus time for base case and voidage management injection scenarios.

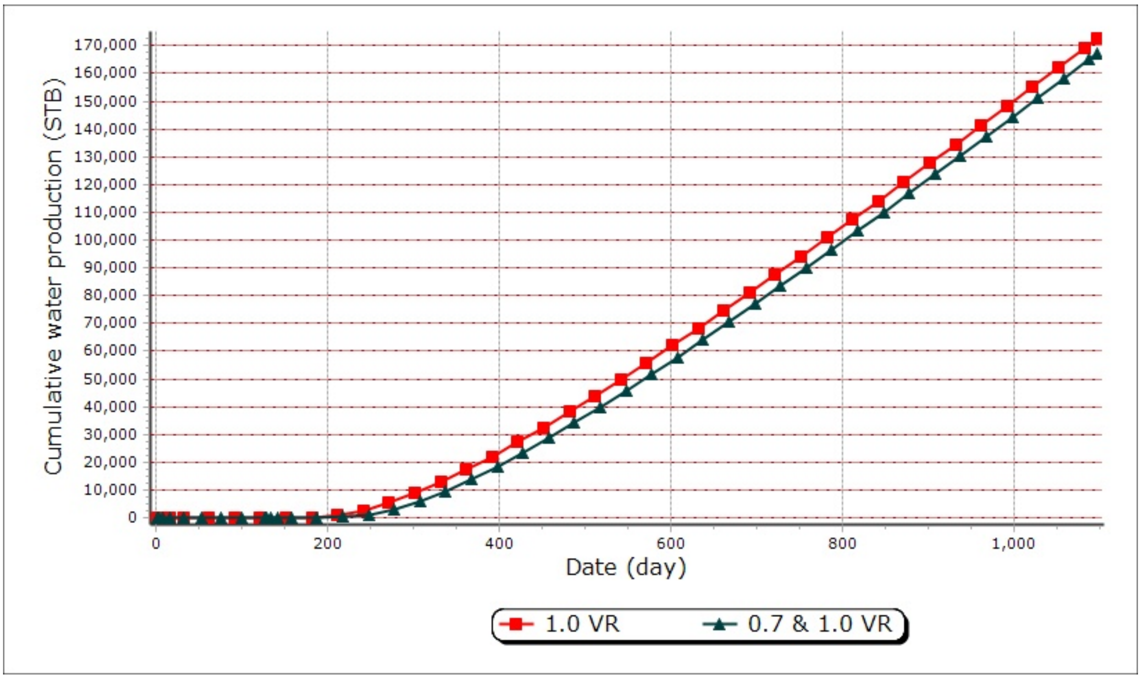


Figure 2. 31: Plot of cumulative water production versus time for base case and voidage management injection scenarios.

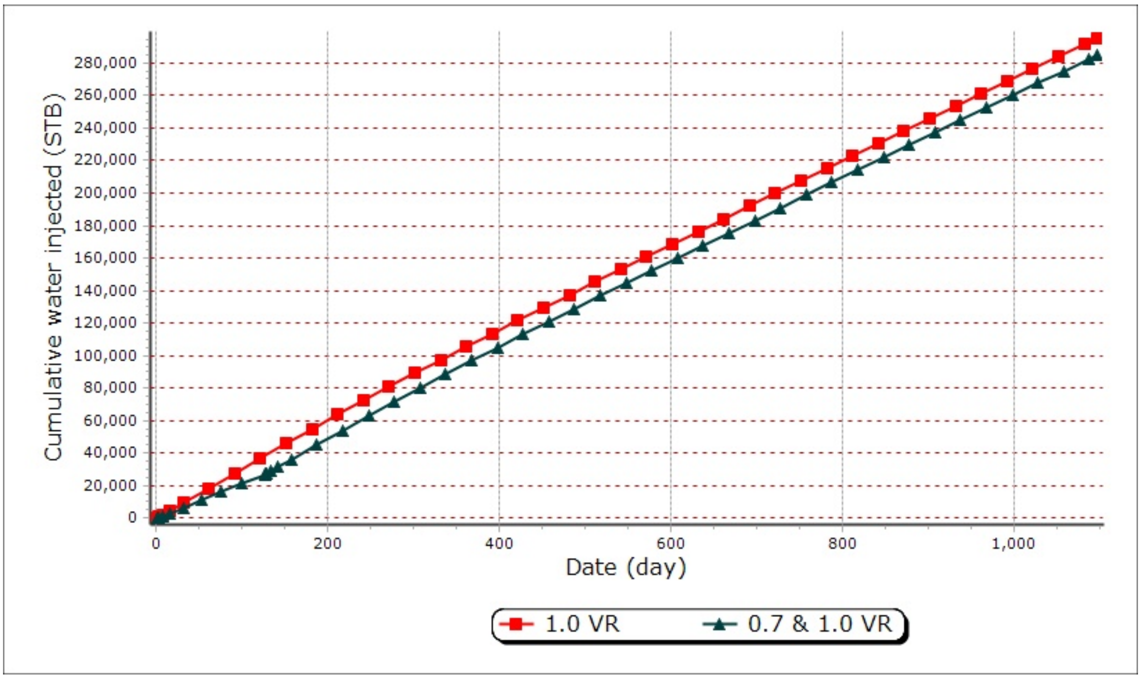


Figure 2. 32: Cumulative injected water versus time for base case and voidage management injection scenarios.

2.4 Results and discussion

In general, the techniques reviewed can be classified in to two groups. The first group comprises those methods that addressed the determination of future oil reserves or future oil recovery based on past production data. The second group of techniques, that analyse the injection well's history, are used to monitor the performance of the injection well.

The first group includes the Cut-cum, *WOR* and the X-plot. To some extent, and by selecting the most appropriate section of the plots, they were accurate enough to predict future cumulative oil production (the Cut-Cum & *WOR* plots) or future oil recovery (the X-plot). This study indicated that Cut-Cum plot predicted better than the *WOR* plot but the results were very sensitive to the polynomial fitted to the curve. By contrast, a simple linear polynomial was employed for extrapolating the *WOR* plot. The most accurate estimation was obtained from X-plot. This can be used to generate an effective permeability plot that, unlike the laboratory curves, is a composite curve that includes the reservoir geometry, heterogeneity, and operational conditions of the field, along with the displacement characteristics of the fluids [17]. Note that, in order to generate the X-plot, f_w should be transformed to X values by Equation 2.12 and the extrapolation is only valid for f_w higher than 50%. The proposed technique in the X-plot is based on actual performance of a waterflood project.

It should be noticed that one major assumption in the techniques used for estimating future performance of the waterflood was that the operating method will remain relatively unchanged. Any variations in operational procedure such as change in the injection or production strategy, infill drilling or shutting down wells, will result in a shift of actual performance, and should be considered in updating the plots [11]. In this study, we used the reservoir simulation injection and production history. Data preparation for forecasting is another important factor for good estimation. The input data set should be cleaned from any noise and they should represent the current condition of the waterflood system.

The only technique that belongs to the second group is the Hall plot. In this study, the Hall plot provided good information about the performance of the injection well performance such as injectivity index or change in the skin, but it could not give any information about the overall performance of waterflood, in terms of improvements in the oil recovery or a decrease in water production.

One of the main questions that this study sets out to answer is: are these techniques able to determine the optimum injection parameters in order to improve waterflood performance?

Unfortunately, the answer is no. Those methods in the first group showed that *WAM* improved the oil recovery from the reservoir but there is no way to find how this improvement was achieved. In the case of the Hall plot, it shows the change in the injection parameters from base case to *WAM* but we could not determine the influence of this change on waterflood efficiency.

The only way that we can use these methods for defining an algorithm for water flood management, such as determining the total amount of water injection or water allocation between injectors, is to use the plot obtained from a previous successful flood project as a type curve, in order to control injection for a new project that has the same flood characteristics as the oil project. In this case, the Hall plot can be a good candidate, as it directly addresses the waterflood parameters, such as injection rate or injection pressure. For example, if an X-plot of a water injection project shows a good oil recovery, the Hall plot of the associated injectors can be employed for the injection control of another reservoir with the same properties as the current one.

The illustrated workflow for the reservoir voidage management shows a significant increase in oil recovery. Not only did oil production increase, water production reduced and this happened with less injection volume of water. High volume injection of water can maintain reservoir pressure but it also causes early water breakthrough. This can cause a poor well outflow performance and reduces the efficiency of the water/oil displacement process. Proper reservoir voidage management can avoid excessive water production and increase oil recovery. This example illustrates that it is not always necessary to maintain the reservoir pressure at the initial reservoir pressure. Defining the suitable reservoir pressure, based on well outflow performance and oil recovery, is a simple approach to define a voidage management strategy. Water breakthrough time is another important parameter that can help to develop a suitable voidage plan for the reservoir.

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Chapter 3– Producer-Injector Inter-well Connectivity Measurement; Statistical Approach

3.1 Producer-injector inter-well connectivity

Numerous studies have concluded that connectivity is one of the most important factors controlling the success of improved oil recovery processes [1]. Inter-well connectivity evaluation determines how effectively two wells are connected to each other (Figure 3.1). This can provide useful information on reservoir heterogeneity, identify flow barriers and conduits and provide tools for reservoir management and production optimization thus it leads to better waterflood management [2].

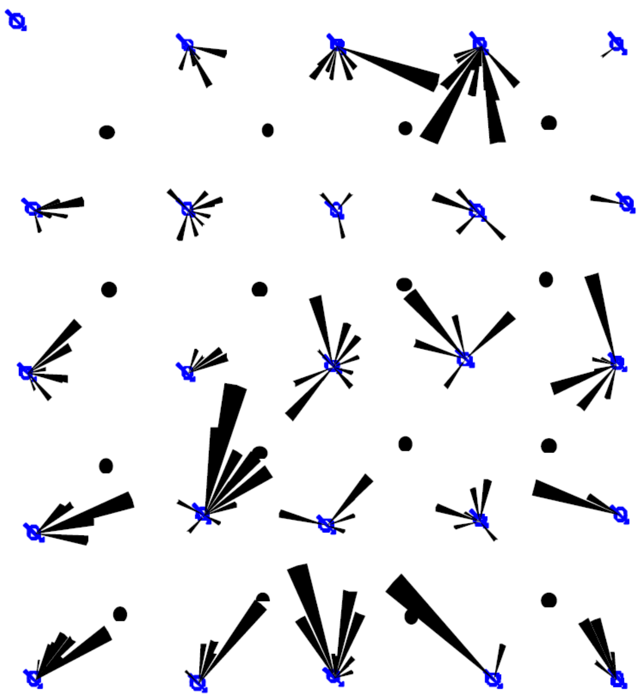

Figure 3. 1: An example of calculated inter-well connectivity between injectors (blue circles with arrows) and producers (black circles) [3].

Typically, reservoir description and characterization, together with observation of injection and production rates, are used to determine the influence of each injector on producers. Analysis of injection and production data can be particularly useful for determining connectivity between well pairs, since flow rates are among the most commonly made measurements made during field life [4].

Appropriate determination of this parameter will lead to efficient design of the injection scheme and infill drilling program [5]. Inter-well connectivity measurement can be coupled with the performance of the producers, to manage the allocation of water between injection wells. This will lead to supporting good oil producers and decreasing the water production in producers with high water cut.

3.2 Inter-connectivity measurement

Modelling the trend of fluid flow between wells in petroleum reservoirs for proper reservoir management and rate allocation is a complex problem. This is because of the non-linear nature of the interaction between parameters such as pressure, temperature, chemical composition of the fluid and reservoir heterogeneity [6].

Probably the most widely available source of data for waterflood management is the monthly welltest production and injection rates. Useful and valuable information can be obtained by proper analysis of such data. In general, reservoir description combined with observation of production and injection history is frequently used to determine the connection between producers and injectors [7].

Different techniques introduced to quantify communication between wells in the reservoir. Most of these methods are based on the analysis of injection and production history data. In this chapter, we will review and evaluate the common statistical methods that have been developed for analysing the injection and production data.

1. Spearman Rank Correlation
2. Multi-Linear Regression (*MLR*)
3. Capacitance Resistive Model (*CRM*)

3.2.1 Spearman rank correlation coefficient

The Spearman rank correlation coefficient [8] (r_S) is a function of the sum of the square of the difference of the two rankings for each observation and the number of the observations [9]:

$$r_s = 1 - \frac{6}{n(n^2-1)} \sum_{i=1}^{n} d_i^2 \qquad \text{(Equation 3.1)}$$

where d_i is difference between the rankings of the ith observations and n is the number of observations. Because r_s is a correlation coefficient, it has a value between *-1* and *+1*. If there is a perfect positive correlation, all the differences will be zero and r_s equals +1. If there is a perfect negative correlation, in which the low-ranking observation in one classification corresponds to the high-ranking observation in the other, value of the r_s will be *-1*. If the two ranking sets are independent, this correlation coefficient will be zero.

The Spearman rank correlation coefficient is a quick, simple, and powerful test of the existence of the association between variables, regardless of the population distribution from which the samples are drawn [10].

The Spearman rank correlation coefficient is thus an ideal tool for investigating the correlation between production and injection rates in a reservoir to determine the communication between injectors and producers. An increased correlation indicates a greater connection between an injector and a producer [11].

3.2.2 Multi-linear regression (MLR)

Multi-linear Regression views the reservoir as a system that processes a stimulus (injection) and returns a response (production). In a waterflood system with multiple injectors and producers, the effect of this input/output on the reservoir will depend on the orientation and location of each well. This technique uses different statistical approaches, based on constrained multi-linear regression, to infer connectivity [7].

The liquid (oil and water) production rates and the water injection rates for every well that makes up the waterflood system are the main input data. The gas production rate is not included in the analysis; hence periods with no significant free gas production must be selected for analysis with this technique.

Production and injection rates in reservoir volumes are used as input to derive the coefficients for the equation used to estimate the production rate. Two different systems will be considered [4]:

1. **An unbalanced system:** injection rate and production rate are different:

$$q_j(t) = \beta_{0j} + \sum_{i=1}^{I} \beta_{ij} i_i(t)$$

$$(j = 1,2,\ldots, N) \qquad \text{(Equation 3.3)}$$

where $q_j(t)$ is the production rate, $i_i(t)$ is the injection rate, N is the total number of producers and I is the number of injectors. This equation states that, at any time, the total production rate at well j is a linear combination of the rates of every injector plus a constant term, β_{0j}. The factors β_{ij} are the weighting factors and the constant term β_{0j} accounts for the unbalance [4].

Jensen et al. present the solution procedure for the *MLR* problem. This involves minimising the variance between the actual production rates and the estimated one $(\hat{q}_j)$. This will lead to the following I linear equations [4]:

$$\begin{pmatrix} \sigma_{11}^2 & \cdots & \sigma_{1I}^2 \\ \vdots & \ddots & \vdots \\ \sigma_{I1}^2 & \cdots & \sigma_{II}^2 \end{pmatrix} \times \begin{pmatrix} \beta_{1j} \\ \vdots \\ \beta_{Ij} \end{pmatrix} = \begin{pmatrix} \sigma_{1j}^2 \\ \vdots \\ \sigma_{Ij}^2 \end{pmatrix} \qquad \text{(Equation 3.4)}$$

This can be solved by standard means. The constant β_{0j} is given by:

$$\beta_{0j} = \bar{q}_j - \sum_{i=1}^{i=I} \beta_{ij}\bar{\imath}_i \qquad \text{(Equation 3.5)}$$

in which $\bar{q}_j$ and $\bar{\imath}_i$ are the average values of production and injection rates.

2. **A balanced system (*BMLR*):** in which the total field injection rate approximately equals the total field production rate. The system can be described by [4]:

$$q_j(t) = \sum_{i=1}^{I} \beta_{ij} i_i(t) \qquad (j= 1,2,\ldots N) \qquad \text{(Equation 3.6)}$$

Again the production rate of producer j is a linear combination of the injection rates of its associated injectors. In this case, there is no constant term and the average balanced condition can be given by

$$\bar{q}_j(t) = \sum_{i=1}^{I} \beta_{ij} \bar{i}_i(t) \qquad (j= 1,2,\ldots N) \qquad \text{(Equation 3.7)}$$

Inclusion of the constraints of the *BLMR* condition, can be achieved by employing the Lagrange multiplier (μ_j) in a similar manner to kriging. Then the following term should be minimised [4]:

$$Var(\hat{q}_j - q_j) - 2\mu_j \left[\bar{q}_j - \sum_{i=1}^{I} \lambda_{ij} \bar{i}_i \right] \qquad \text{(Equation 3.8)}$$

resulting in the final set of the equation:

$$\begin{pmatrix} \sigma_{11}^2 & \dots & \sigma_{I1}^2 & \bar{\iota}_1 \\ \vdots & \ddots & & \vdots \\ \sigma_{I1}^2 & \dots & \sigma_{II}^2 & \bar{\iota}_I \\ \overline{\iota_1} & \dots & \overline{\iota_{I1}} & 0 \end{pmatrix} \times \begin{pmatrix} \beta_{1j} \\ \vdots \\ \beta_{Ij} \\ \mu_j \end{pmatrix} = \begin{pmatrix} \sigma_{1j}^2 \\ \vdots \\ \sigma_{Ij}^2 \\ \bar{q}_j \end{pmatrix} \quad \text{(Equation 3.9)}$$

Equation 3.9 can be solved for β_{Ij} by standard means. Sets of *I+1* equations with *I+1* unknowns must be solved for each producer, β_{ij}, providing a quantitative expression of the connectivity between injector and each producer [4].

3.2.3 Capacitive resistive model (CRM)

The *MLR* technique does not address the time lag between injection wells and production wells directly. This should be done before *MLR* analysis by filtering the input data. It also neglects the effect of production wells on each other. Yousef et al. (2005) [12-14] introduced new analytical method called the capacitive resistive model (*CRM*) that can quantify inter-well connectivity and the degree of fluid storage available between injection and production wells [15].

Like *MLR*, *CRM* is a non-linear multivariate-regression analysis in which [16], the reservoir is considered as a system that converts inputs (injection rates) into outputs (production rates) [17]. Two coefficients are determined for each injector-producer pair: one parameter (weight) quantifies the connectivity and the other (time constant) quantifies the degree of fluid storage between the wells [18]. It can therefore provide good information about the preferential transmissibility trends present within a waterflood as well as the presence of flow barriers [12].

Mathematical development: The *CRM* considers the total mass balance of the injection and produced fluid along with compressibility. A single injector-producer well pair in a drainage volume is the simplest case. The governing material balance differential equation at reservoir conditions is introduced by the following equation [13]:

$$C_t V_p \frac{d\bar{P}}{dt} = i(t) - q(t) \qquad \text{(Equation 3.10)}$$

In which C_t is the total compressibility; V_p is the drainage pore volume; $\bar{P}$ is the average pressure in V_p; *i(t)* is the total injection rate and *q(t)* is the total production rate. According to this equation, at any time, the total rate of mass depletion from the drainage volume is related to the rate of change of the average pressure within the volume [13].

In Equation 3.10, it has been assumed that the total compressibility of the reservoir is small and constant and there is no fluid transfer out of or into the volume V_p. It can also be derived from a spatial integration of the diffusivity equation, under the same assumption. With $i(t) = 0$ this equation is used by Walsh and Lake to describe primary depletion. The equation is also used to describe the flow of electrical current in a resistor-capacitance network, which has the same form, hence the term capacitance in the description [14].

Equation 3.11 introduces the linear productivity index model that helps to describe the the system, based entirely on the rates [13]:

$$q(t) = J\left(\bar{P}_r - P_{wf}\right) \qquad \text{(Equation 3.11)}$$

where p_{wf} and J are the flowing bottom hole pressure (*BHP*) and productivity index of the producer, respectively. Equation 3.11 assumes stabilized flow, which is unlikely to be accurate in conditions where rates are constantly changing. However, the productivity index (and equivalent alternative definitions) is almost universally applied in describing well performance, in practice [14].

Eliminating the average pressure between Equations 3.10 and 3.12 give:

$$\tau \frac{dq}{dt} + q(t) = i(t) - \tau J \frac{dP_{wf}}{dt} \qquad \text{(Equation 3.12)}$$

where τ is the "time constant" of the drainage volume, and is defined by:

$$\tau = \frac{C_t V_p}{J} \qquad \text{(Equation 3.13)}$$

The differential equation (Equation 3.13) being derived from the linear equation of productivity index and material balance equation, respects the following assumptions [19]:

1. Slightly compressible fluid
2. Instantaneous equilibrium
3. Immiscible phase
4. Constant temperature
5. Constant productivity

The solution to the above equation is [13]:

$$q(t) = q(t_0)e^{-\left(\frac{t-t_0}{\tau}\right)} + e\frac{-t}{\tau}\int_{t_0}^{t} e^{\frac{\xi}{\tau}}\frac{1}{\tau}i(\xi)d\xi + e\frac{-t}{\tau}\int_{t_0}^{t} e^{\frac{\xi}{\tau}}Ji(\xi)\frac{dp_{wf}}{d\xi} \qquad \text{(Equation 3.14)}$$

This means that the production rate is affected by [14]:

1. Primary production
2. Contribution from injection
3. Change in bottomhole flowing pressure

Sayarpour et al. [20] introduced analytical solutions for the fundamental differential equation of the *CRM*, based on superposition in time and presented these solutions for three different reservoir control volumes: 1) drainage volume of each producer (*CRMP*), 2) volume of the entire field or tank model (*CRMT*), and 3) drainage volume between each injector-producer pair (*CRMIP*) [15].

1. **One time constant for each producer (*CRMP*):** For a pattern of *I* number of injectors and *N* number of producers, Figure 3.2 represents the in-situ volumetric balance over the effective pore volume of a producer. Sayarpour et al. derived analytical solutions for two cases: a linear variation of bottom-hole flowing

pressure (*BHP*), but with stepwise changes in injection rate, and a linear variation of both injection rate and BHP during consecutive time intervals [19].

a) For a case of fixed injection rate of i_i $(\Delta_{tk}) = I^{(k)}{}_i$, and a linear *BHP* variation during time intervals Δ_{tk} , $(k=1,2,\ldots,n)$, by assuming a constant productivity index at time t_n , the total production rate of producer j can be written as [19]

$$q_j(t_n) = q_j(t_0)e^{\frac{-(t_n-t_0)}{\tau_j}} + \sum_{k=1}^{n}\left[\left(\sum_{i=1}^{I} f_{ij}I_{ij}^{k} - J_j\tau_j\frac{\Delta P_{wf,j}^{(k)}}{\Delta t_k}\right)\left(1-e^{\frac{-\Delta t_k}{\tau_j}}\right)e^{\frac{-(t_n-t_k)}{\tau_j}}\right]$$

(Equation 3.15)

where I_{ij}^{k} and $\Delta P_{wf,j}^{(k)}$ represent the injection rate of injector i and the changes in *BHP* of producer j during the time interval t_{k-1} to t_k, respectively. Equation 3.15 effectively assumes that the variation of injection rates is stepwise.

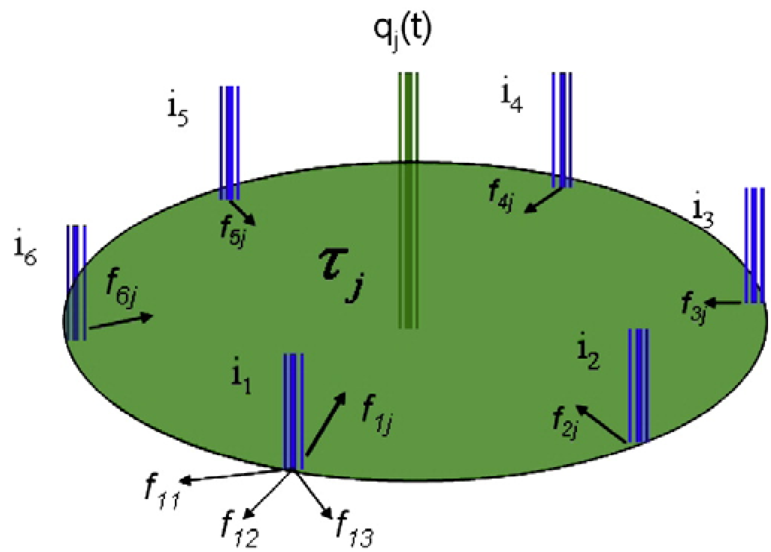

Figure 3. 2: Schematic representation of control volume for producer j, CRMP [21].

b) If there is a linear change between two consecutive injection rates and *BHP* during time intervals t_{k-1} to t_k, at time t_n, the total production rate of producer j can be written as [19]

$$q_j(t_n) = q_j(t_0)e^{\frac{-(t_n-t_0)}{\tau_j}} + \sum_{i=1}^{I}\left[f_{ij}\left(i_i(t_n) - e^{\frac{-(t_n-t_0)}{\tau_j}} i_i(t_0)\right)\right] - \sum_{k=1}^{n}\left[\left(\sum_{i=1}^{I} f_{ij}I_{ij}^{k} - J_j\tau_j\frac{\Delta P_{wf,j}^{(k)}}{\Delta t_k}\right)\left(1-e^{\frac{-\Delta t_k}{\tau_j}}\right)\tau_j e^{\frac{-(t_n-t_k)}{\tau_j}}\right] \quad \text{(Equation 3.16)}$$

2. **One time constant for field (*CRMT*):** A reservoir may be represented by a single producer and a single injector, as a tank. This is achieved by combining all production and injection rates (Figure 3.3). Hence, the *CRMP* solution can be applied by equating f_{ij} to unity to arrive at the tank solution (designated as *CRMT*). Therefore [19]:

$$q_F(t_k) = q_F(t_0)e\frac{-(t_n-t_0)}{\tau_F} + \sum_{k=1}^{n}\left[I_F^k e^{-\left(t_n-t_k\right)}\left(1-e^{\frac{-\Delta t_k}{\tau_F}}\right)\right] \quad \text{(Equation 3.17)}$$

in which $I_F(t)$ and $q_F(t_k)$ are the total field injection and production rates. τ_f is the field-time constant and, hence, represents the field average properties [19].

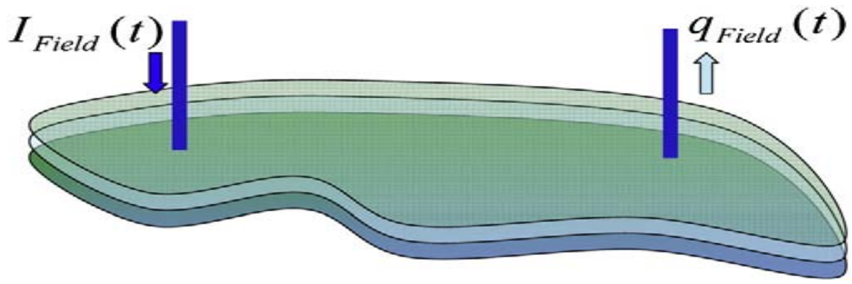

Figure 3. 3: Schematic representation of a field with one injector and one producer, CRMT [21].

CRMT does not account for the variation of bottom-hole pressures in individual wells, therefore only two system parameters, initial production rate and field time constant, are sought [19].

3. **One time constant for each injector-producer (*CRMIP*):** The volumetric balance in reservoir conditions over the affected pore volume injector/producer pair, *ij*, is shown in Figure 3.4. Similar boundary conditions as those discussed above for *CRMP*, may be used to obtain analytical solutions, by assuming a linear variation of a) *BHP*, while the injection rate is changing in steps, and b) both injection rate and *BHP* during consecutive time intervals [20].

a) The total production rate of producer *j* for the case of fixed injection rate and a linear BHP variation during time intervals, at time t_n, can be written as [20]:

$$q_j(t_n)=\sum_{i=1}^{I} q_{ij}(t_0)e^{\frac{-(t_n-t_0)}{\tau_{ij}}}+\sum_{i=1}^{I}\left(\sum_{k=1}^{n}\left[\left(\sum_{i=1}^{I} f_{ij}I_i^k-J_{ij}\tau_{ij}\frac{\Delta P_{wfj}}{\Delta t_k}\right)\left(1-e^{\frac{-\Delta t_k}{\tau_j}}\right)e^{\frac{-(t_n-t_k)}{\tau_{ij}}}\right]\right)$$

(Equation 3.18)

b) For a case of linear change between two consecutive injection rates and producer's *BHP* during time interval Δt_k , the total production rate of producer *j* at time t_n can be written as [20]:

$$q_j(t_n)=\sum_{i=1}^{I} q_{ij}(t_0)e^{\frac{-(t_n-t_0)}{\tau_{ij}}}+\sum_{i=1}^{I} f_{ij}\left[i_i(t_n)-e^{\frac{-(t_n-t_0)}{\tau_{ij}}}i_0(t_0)\right]$$
$$-\sum_{i=1}^{I}\left(\tau_{ij}\sum_{k=1}^{n}\left[\left(\frac{f_{ij}\Delta i_i^k}{\Delta t_k}-J\frac{\Delta P_{wfj}^k}{\Delta t_k}\right)\left(1-e^{\frac{-\Delta t_k}{\tau_{ij}}}\right)e^{\frac{-(t_n-t_k)}{\tau_{ij}}}\right]\right)$$

(Equation 3.19)

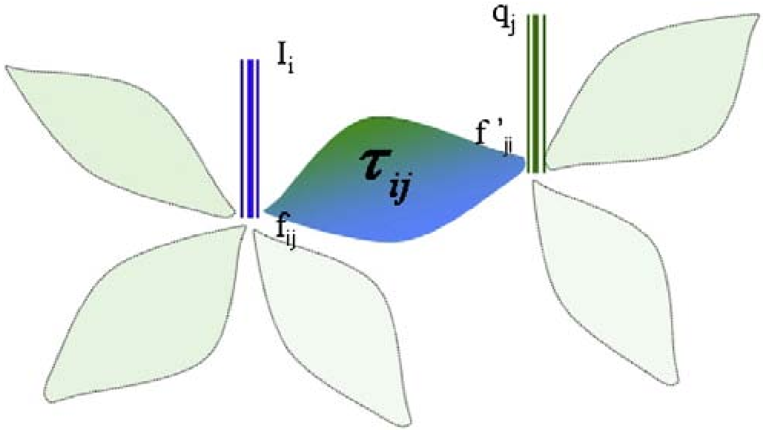

Figure 3. 4: Schematic representation of control volume between each injector/producer pair [21].

3.3 Case study

The same model as used for evaluating the performance of waterflooding monitoring techniques in Chapter 2 of was used to study the application of these techniques for inter-well connectivity measurements. The model has been described in section 2.2.1 (Figure 3.5 and Table 3.1).

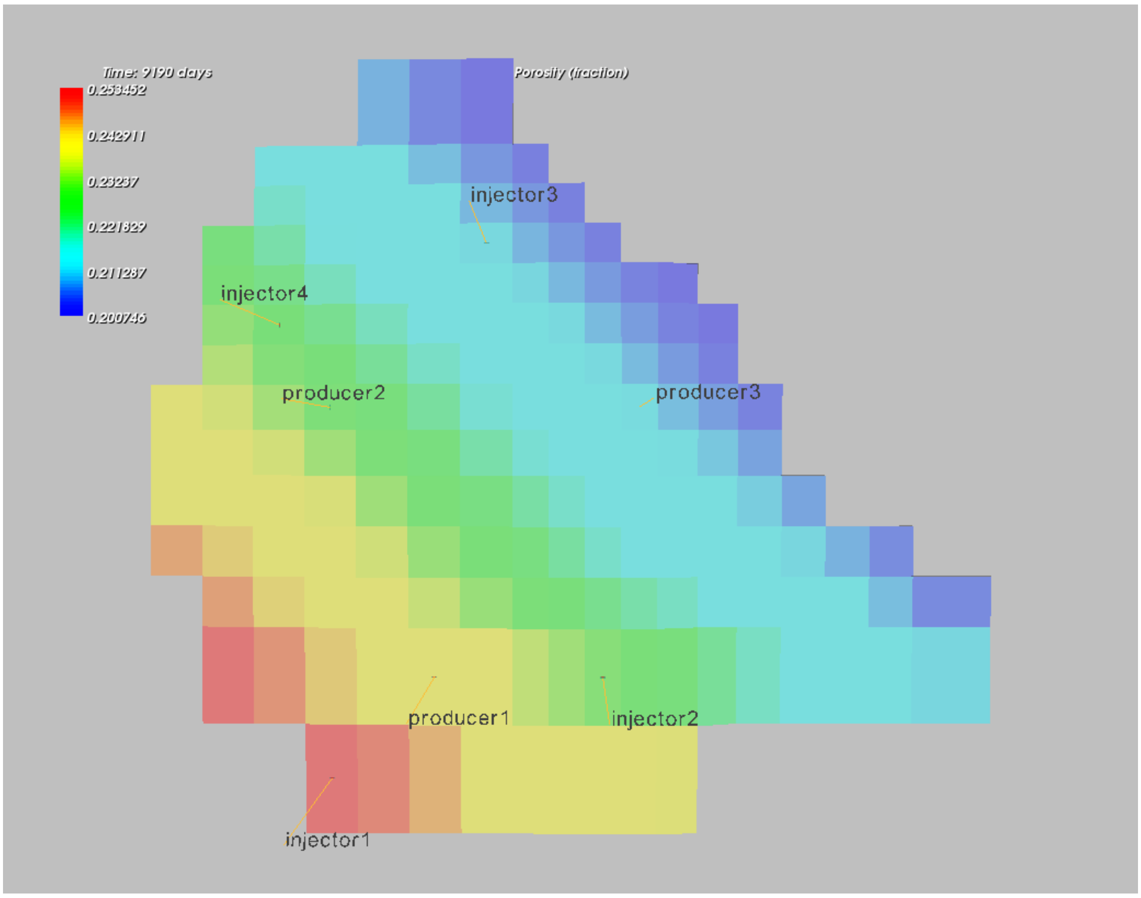

Figure 3. 5: Reservoir model (colours represent the porosity).

Table 3. 3: Properties of the reservoir model.

Size of the cells	20*15*4
Initial oil in place	5.7522×10^{8} STB
Initial pressure	4000 psia
Temperature	100^{o} F
Compressibility	3×10^{6} 1/psi
Permeability	220-320 mD
Porosity (fraction)	0.2-0.25
Oil density	35 API
Bubble point pressure	2054 psia
Gas oil ratio	500 Scf/STB

The reservoir has been producing for 20 years with the pressure support supplied by water injection from the beginning of production. The producers were controlled by a constant surface liquid production rate and injectors set to inject by constant voidage replacement ratio (*VR*). The sum of the all *VRs* is unity i.e. total volume of injected water equals the total produced fluid volume.

Downhole injection and production rates were used as input for all these methods to quantify the connection between wells (Figures 3.6 and 3.7). Then the results for connectivity were combined with the performance of each producer to define the injection allocation factor for each injector.

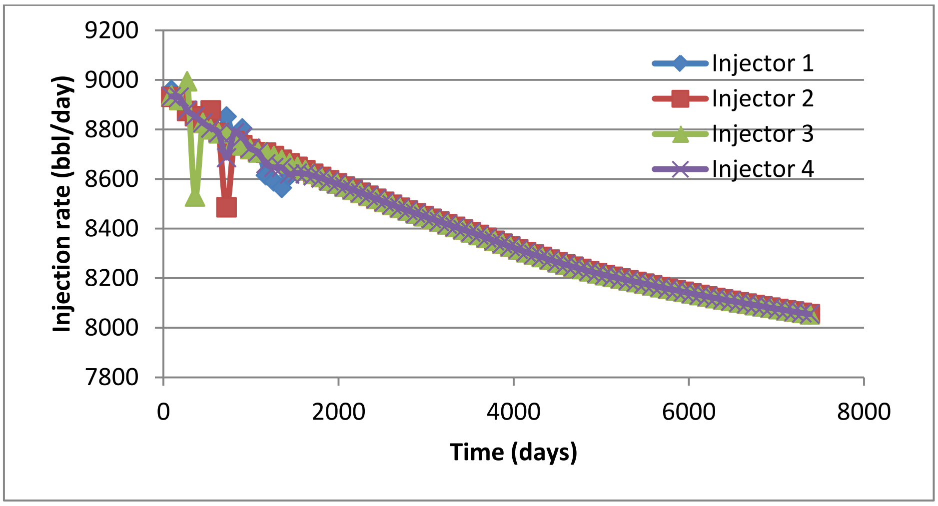

Figure 3. 6: 20-year down-hole injection rate history of all the injectors from the reservoir model.

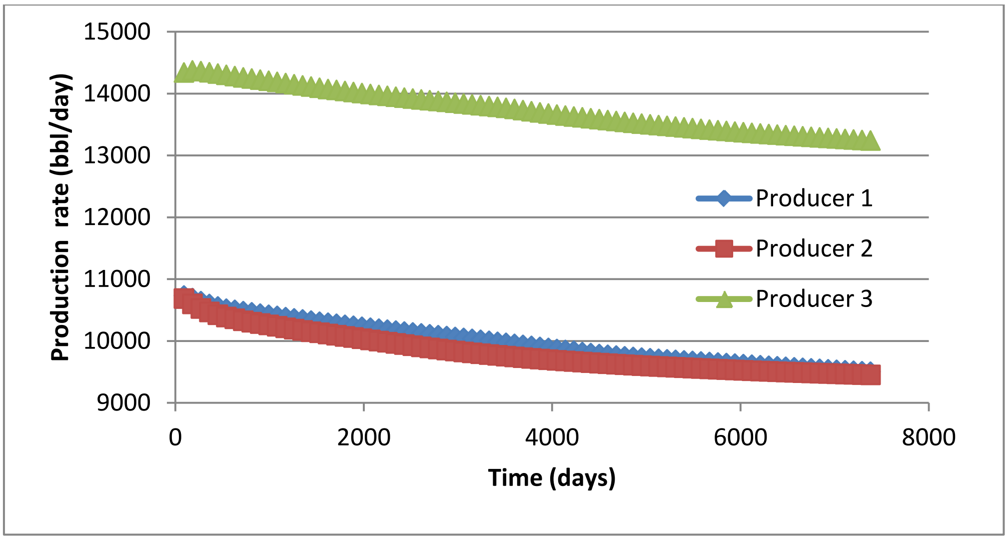

Figure 3. 7: 20-year down-hole production rate history of all the producers from the reservoir model.

3.3.1 Spear Rank correlation coefficient

Data used for this analysis are the injection and production rates in reservoir barrel per day. *SPSS* software is used to calculate the Spearman Rank Correlation coefficient between the each injector/producer pair, to establish the dominant communication trends in the reservoir.

The calculations were based on the liquid production rates of both oil and water. The rates were converted to ranks, and the Spearman rank correlation coefficient was calculated for pairs composed of each injection well and its adjacent production wells. The results are shown in Figures 3.7 and 3.8.

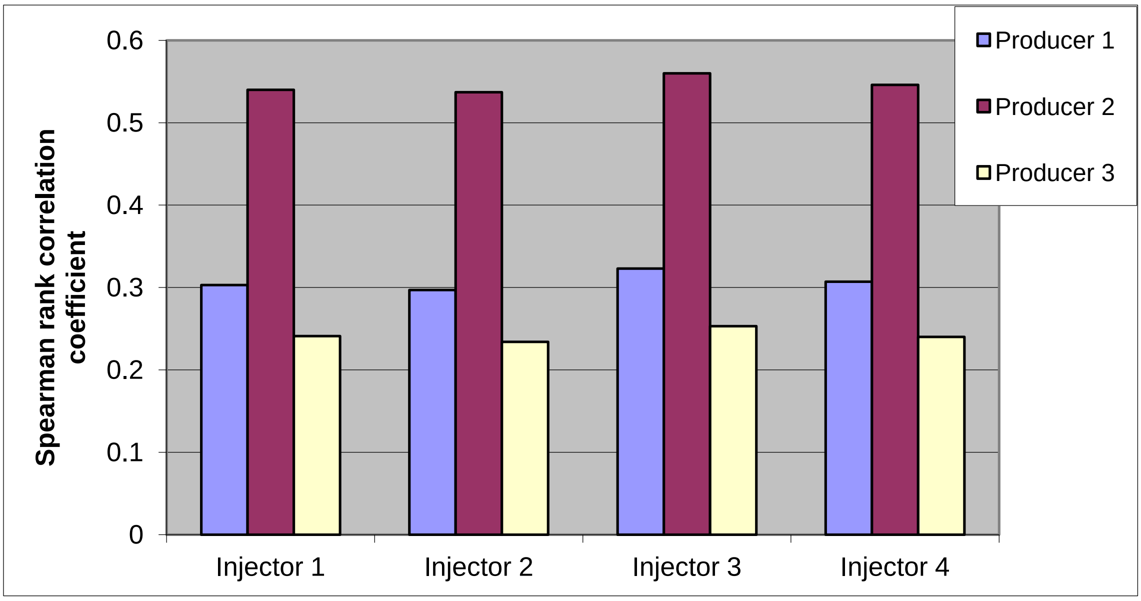

Figure 3. 8: Calculated Spearman correlation coefficients for each injector and its associated producers.

Analysis shows that producer 2 has the highest connectivity with the injectors while producer 3 has the lowest value. And also it can be seen that injector number three has the highest correlation with each producer while injector 2 has the minimum correlation, although the difference is not significant.

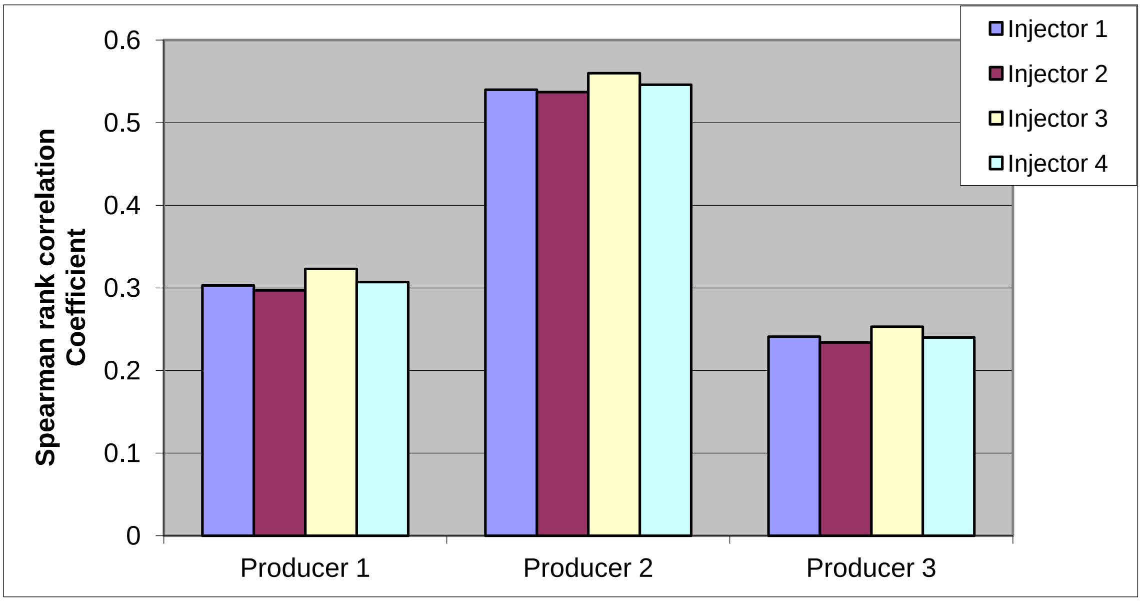

Figure 3. 9: Calculated Spearman correlation coefficients for each producer and its associated injectors.

3.3.2 Multi-linear regression

Balanced *MLR* can also be used to determine the well connectivity, since the injection started at the start of production from the reservoir, while the voidage replacement ratio was one. Equation 3.6 implies that the rate of production from each producer will be a linear combination of the rate of the injectors. Therefore, our reservoir will have 3 producers and 4 injectors; for each producer, the rate of liquid production is:

$$q_j(t) = \beta_{1j} i_1(t) + \beta_{2j} i_2(t) + \beta_{3j} i_3(t) + \beta_{4j} i_4(t) \qquad \text{(Equation 3.19)}$$

The above equation illustrates one of the most important challenges in managing waterflood performance. Is the producer connected to all injectors or just connected to some of them? The answer to this question has a significant effect on the calculated coefficient of Equation 3.19.

For example, let us assume two cases for Producer 1. Case (1) supposes that Producer 1 is connected to all the injectors, while in Case (2) two we assume that it is connected to Injectors 1, 2 and 3 only. Figure 3.10 shows the results of calculated connectivity measurement. It can be seen that there is a significant difference between the calculated coefficients for Injectors 1 and 2, which will affect the measurement of the injection allocation factor.

The superposition approach is proposed to determine which injector is connected to a specified producer. The methodology is as follows:

1. The two injectors closest to the producer are assigned to that producer. The coefficients of the *MLR* equation are then calculated along with a measurement of the estimation error.
2. The next nearest injector is then assigned to the system of one producer and the two injectors. The *MLR* coefficients and the error calculation are repeated.
3. An injector will be assigned to the producer if and only if the calculated error in step 2 is less than in step 1, otherwise it will be considered that the injector is not connected to the producer.

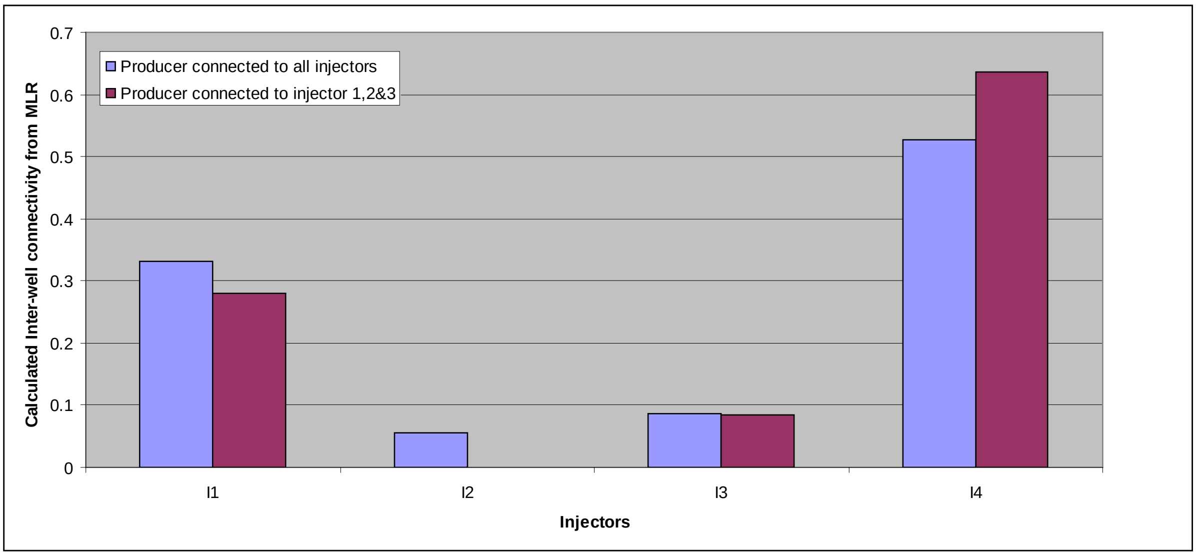

Figure 3. 10: Calculated inter-well connectivity for Producer 1, assuming two different injector-producer combinations.

Figure 3.11 represents the application of this procedure for Producer 1, where the nearest injectors to this producer are Injectors 1& 2.

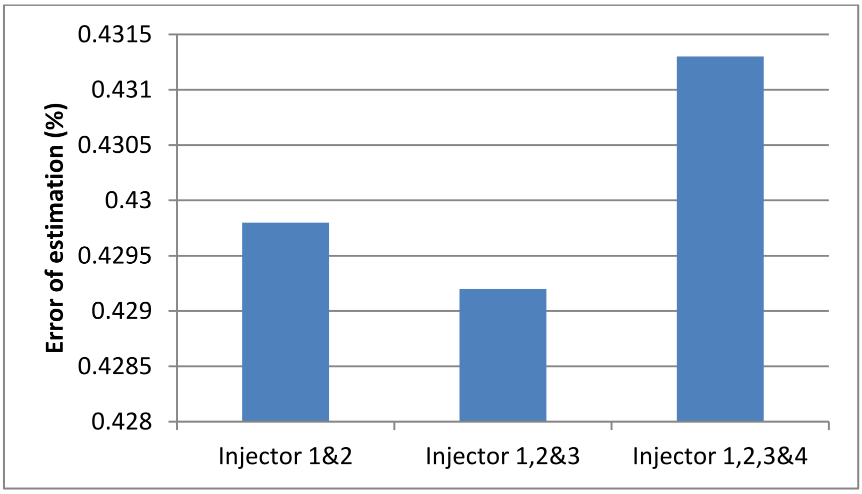

Figure 3. 11: Calculated error of estimation of production rate of Producer 1 for different combinations of injectors.

Adding Injector 3 to the system improves the production rate estimation, while the further addition of Injector 4 increases the error. This implies that Producer 1 is connected to Injectors 1, 2 and 3 but not Injector 4.

The above procedure was applied for all producers. The results are summarised in Figures 3.12 and 3.13 and Table 3.2.

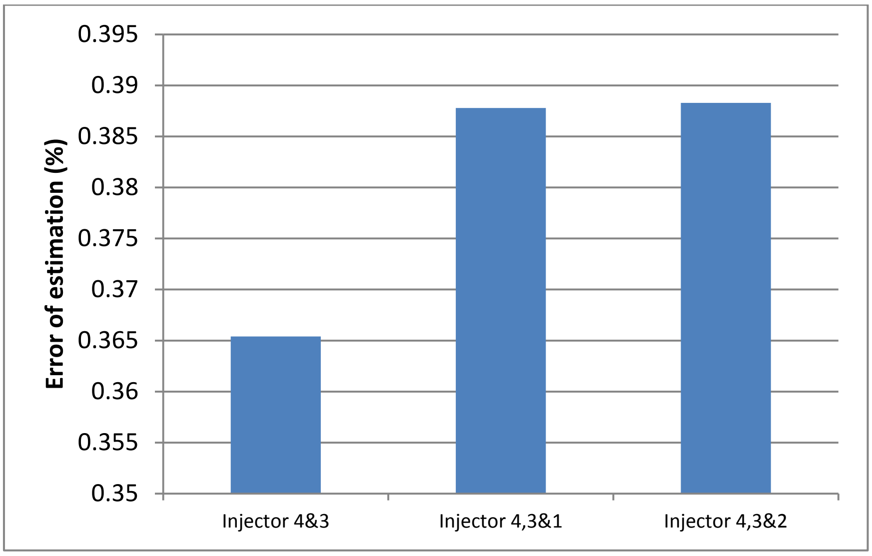

Figure 3. 12: Calculated estimation error of the production rate of Producer 2 for different combinations of injectors.

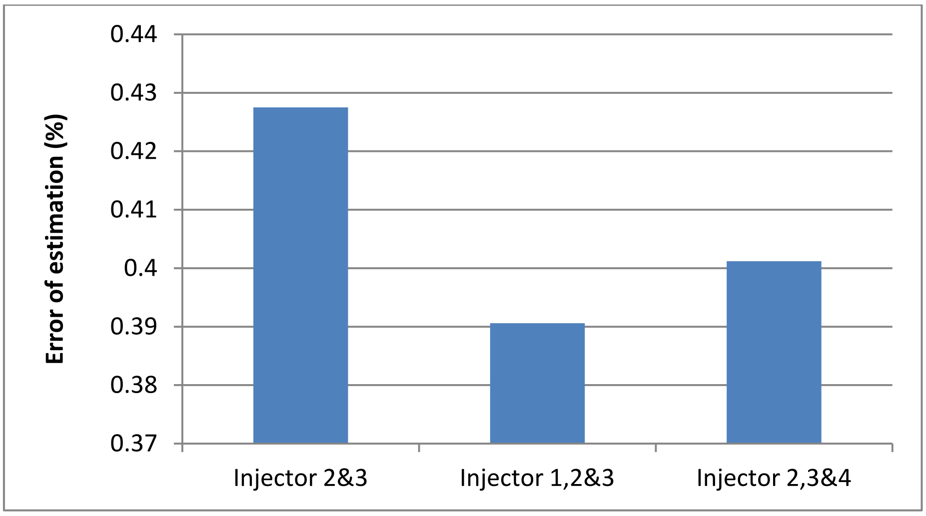

Figure 3. 13: Calculated estimation error of the production rate of Producer 3 for different combinations of injectors.

Table 3. 4: This table shows which injectors are connected to each producer (√ means connected).

	Injector 1	Injector 2	Injector 3	Injector 4
Producer 1	√	√	√	×
Producer 2	×	×	√	√
Producer 3	√	√	√	×

Table 3.2 shows that Producer 1 and Producer 2 are connected to Injectors 1,2 and 3, while Producer 3 is supported by Injectors 3 and 4. The calculated MLR connectivity coefficients for each producer are shown in Figures 3.14, 3.15 and 3.16. These coefficients represent the inter-well connectivity between injectors and producer.

Inter-well connectivity measurement based on *MLR* shows Injector 1 has more connection to Producers 1 and 3 compared to Producers 3 and 2, while Producer 2 is highly affected by Injector 4.

Injector 3 is the only injector that is connected to all the producers but its connection to the production wells is not high, compared to the other injectors.

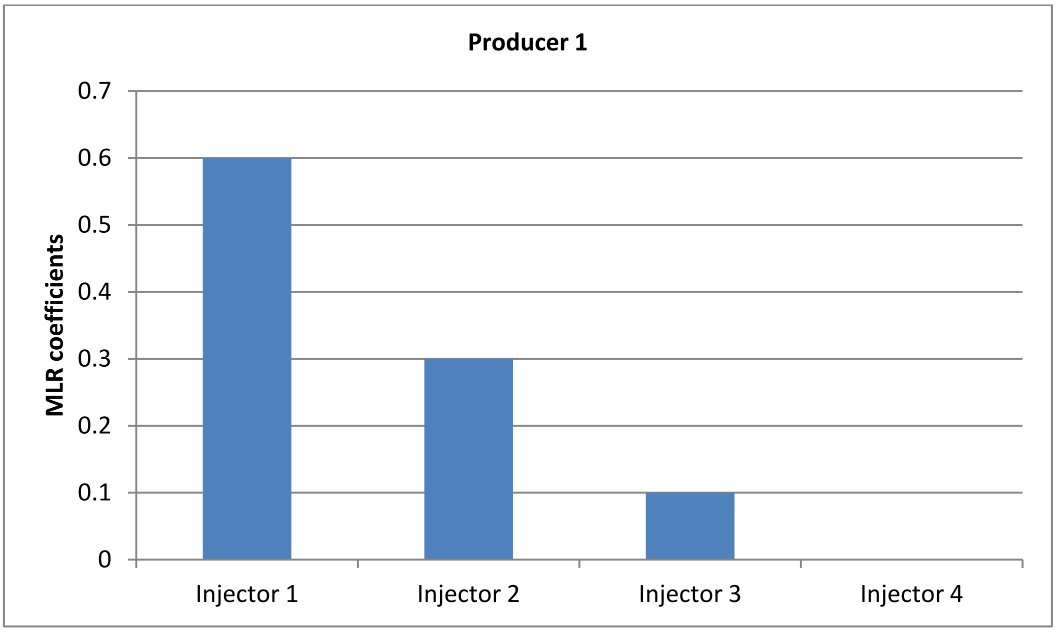

Figure 3. 14: Determined MLR coefficients for the injectors connected to Producer 1.

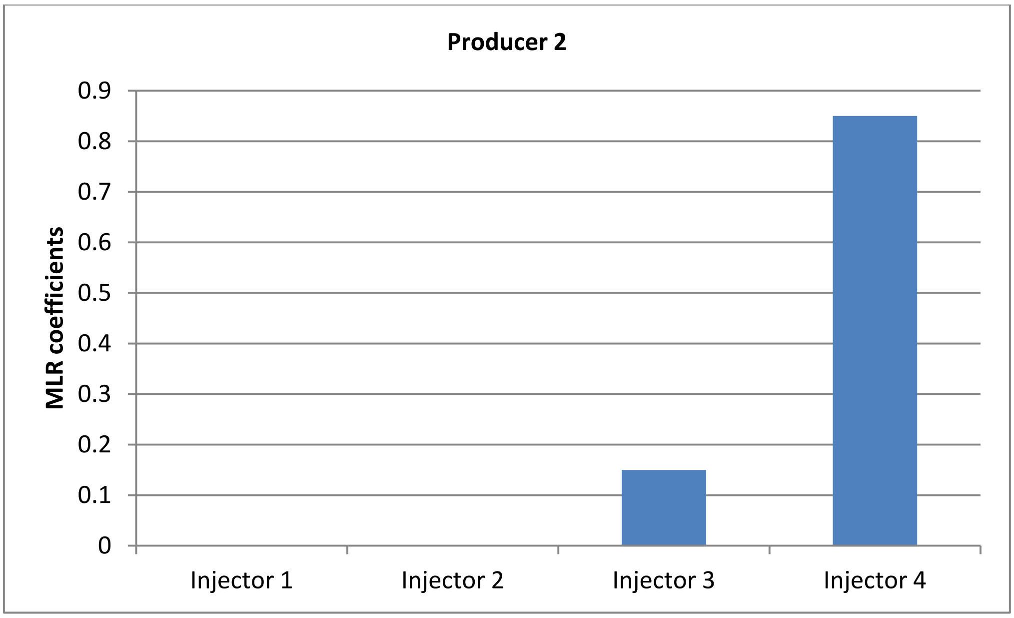

Figure 3. 15: Determined MLR coefficients for the injectors connected to Producer 2.

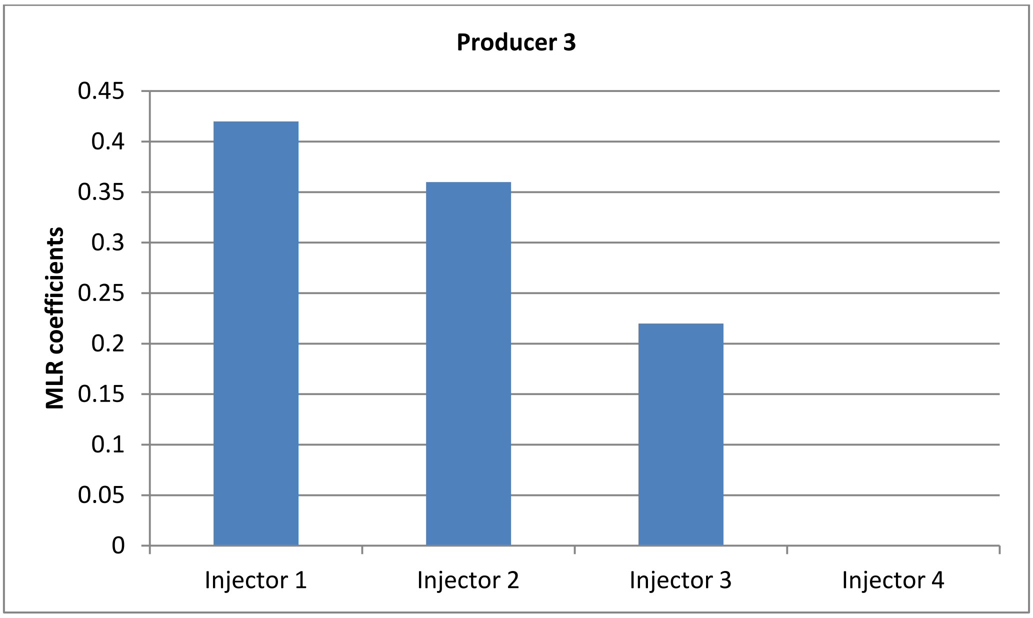

Figure 3. 16: Determined MLR coefficients for the injectors connected to Producer 3.

The following figures (Figure 3.17 to 3.19) represent the estimated production rate using the *MLR* technique for each producer versus the production history obtained from the 20-year reservoir simulation model over the 20-year period. As can be seen, there is a good match between these estimates and the production rate from the simulation.

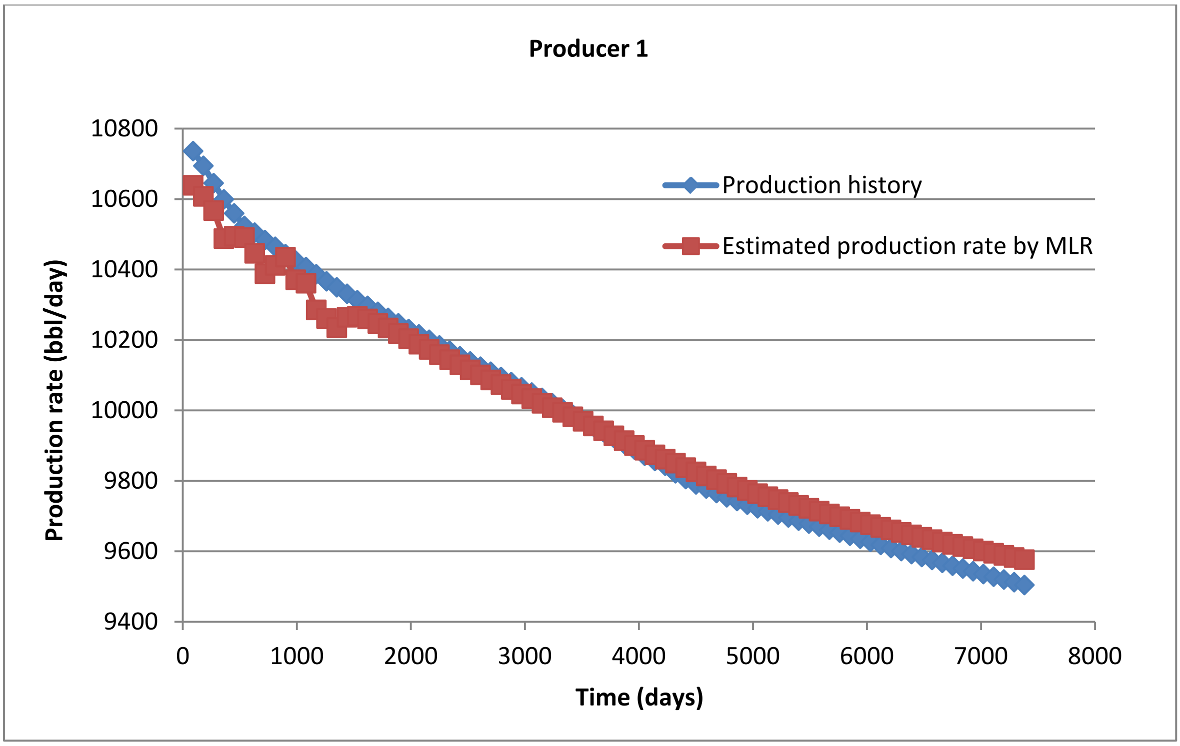

Figure 3. 17: Calculated production rate from MLR versus production history from the reservoir simulation model for Producer 1.

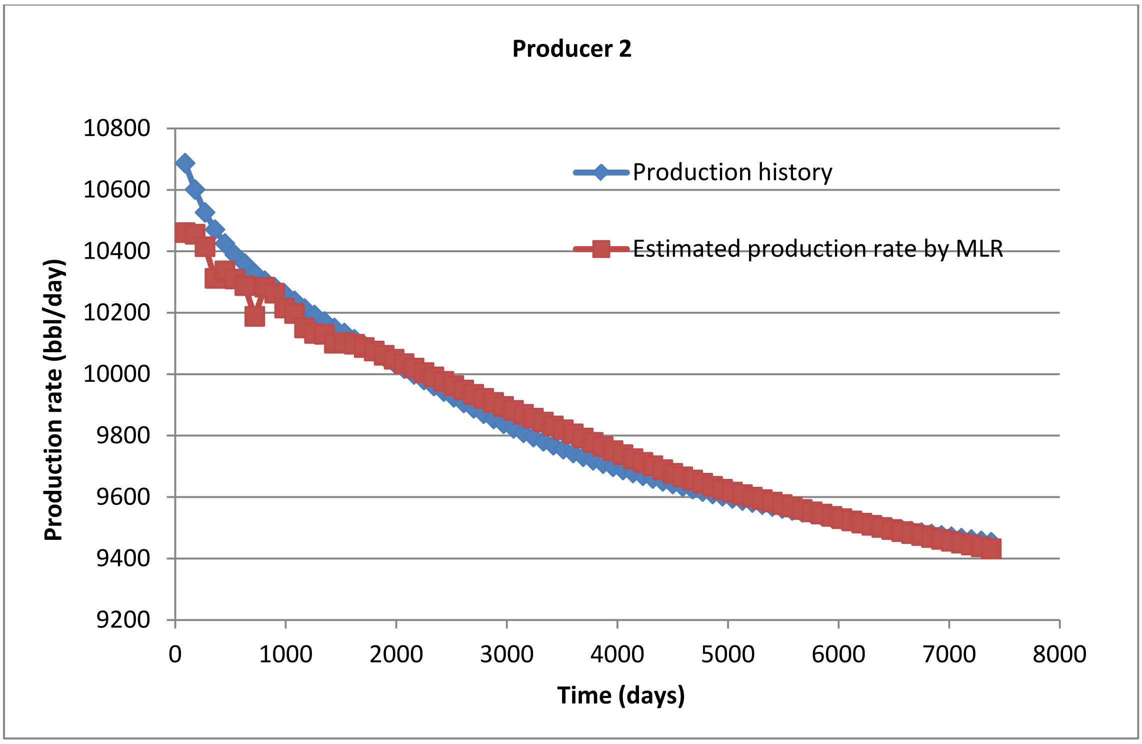

Figure 3. 18: Calculated production rate from MLR versus production history from the reservoir simulation model for Producer 2.

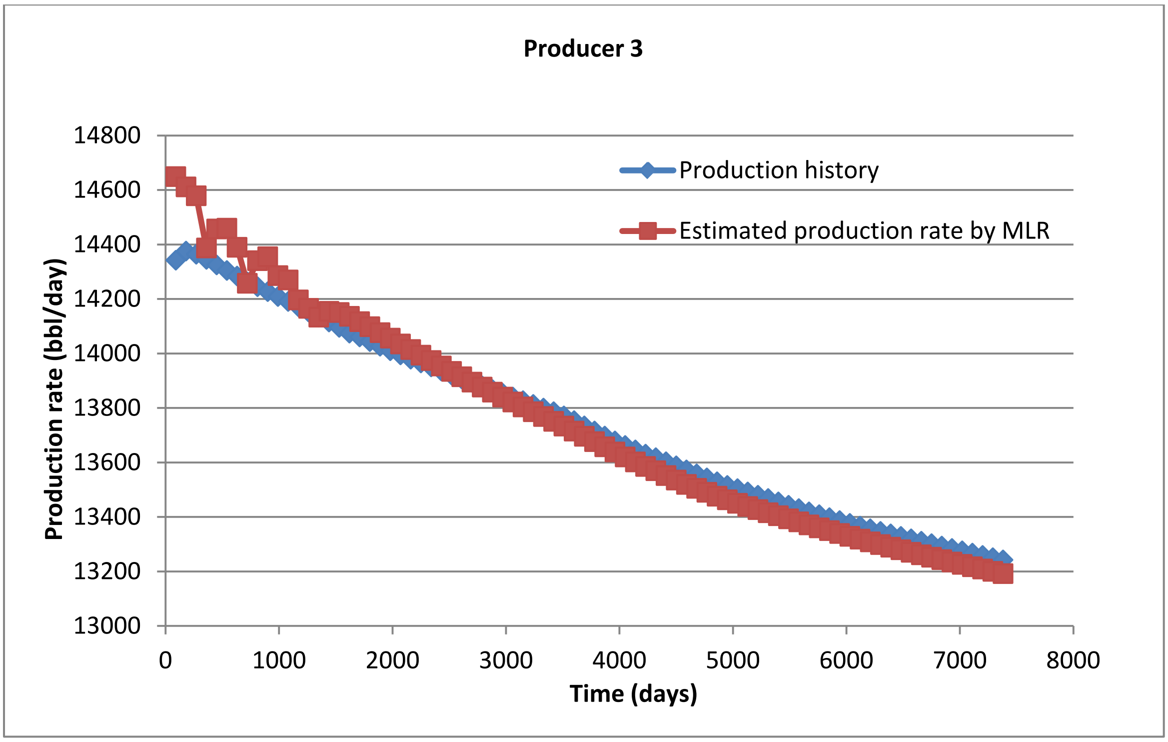

Figure 3.19: Calculated production rate from MLR versus production history from the reservoir simulation model for Producer 3.

3.3.3 Capacitive resistive model

History matching and optimization are two important steps of determining parameters of *CRM* equation. Minimization of the average-absolute error over the total production history is a straightforward way to evaluate model parameters. Since waterflood management by reallocating injected water can be obtained from *CRMP* (Equation 3.18), *CRMP* have been used in this study.

CRMP requires $(I+3)$ number of model parameters for each producer: f_{1j} , f_{2j} ,…, $f_{I,j}$, j , $q_j\ (t_o\ )$ and J_j ; hence, application of *CRMP* requires a minimum of $P\times(I+3)$ injection and production data points. The average absolute of the error for each of the producers can be evaluated and the sum of these errors becomes the objective function [13].

An optimization procedure is required to determine the optimum solution of f_{ij} and τ_j. In the optimization, τ_j s are set to be the free parameters and the objective function is to minimize the squared errors between measured production rates and those generated by Equation 3.18. For a given set of τ_j s Equation 3.18 becomes linear and therefore, *MLR* is used to determine f_{ij}. After iterating in τ_j s, the optimum set of f_{ij} is obtained. The optimum τ_j and f_{ij} are therefore obtained at the end of this procedure. Relying as it does on linear regression, this procedure, allows us to use the error estimates of the weights based on *MLR*[13].

The procedure employed to find which injector is connected to which producer similar to that was used for the *MLR* problem. Table 3.2 shows that the *CRMP* connectivity results are similar to those obtained for *MLR*. Figures 3.20 to 3.22 present the calculated f_{ij} for each producer.

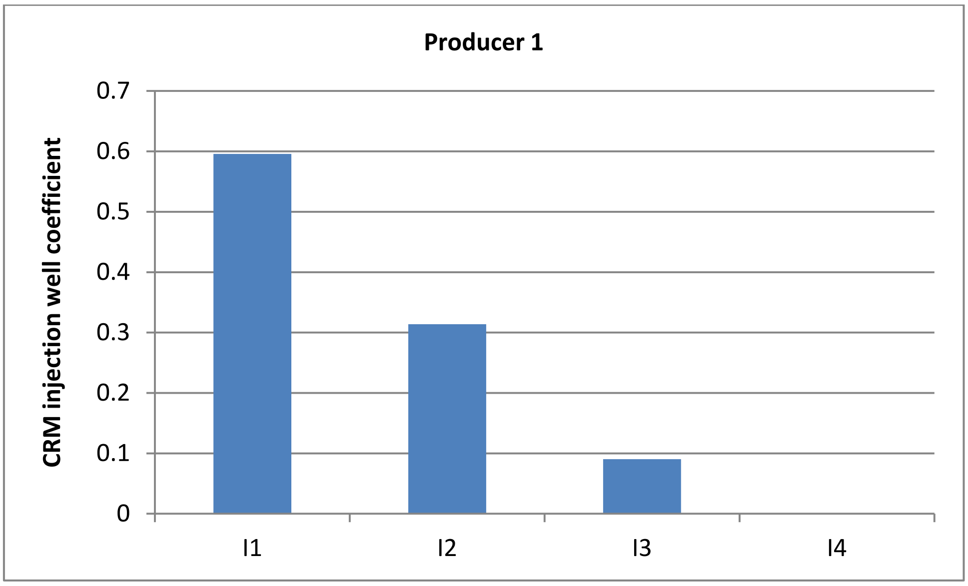

Figure 3. 19: Calculated values of f_{ij} of the CRM equation for Producer 1.

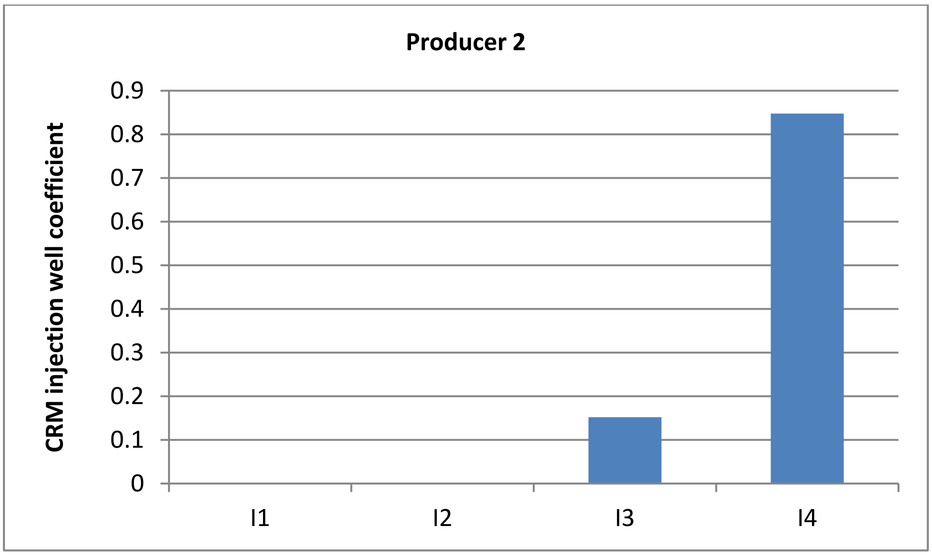

Figure 3. 20: Calculated values of f_{ij} of the CRM equation for Producer 2.

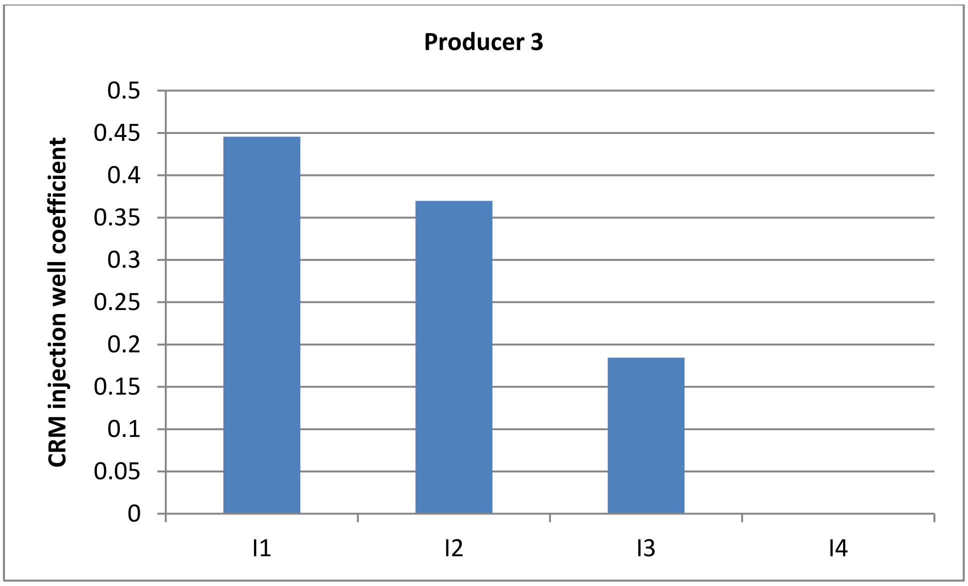

Figure 3. 21: Calculated values of f_{ij} of the CRM equation for Producer 3.

The determined values of the time constant for each producer can be seen in Figure 3.23. Based on values of τ_j, we can say Producer 3 has the highest drainage volume compared to the other producers and Producer 2 has the lowest one. Figure 3.24 that shows the top view of the drainage volume of each producer qualitatively confirms the analysis of the τ_j values.

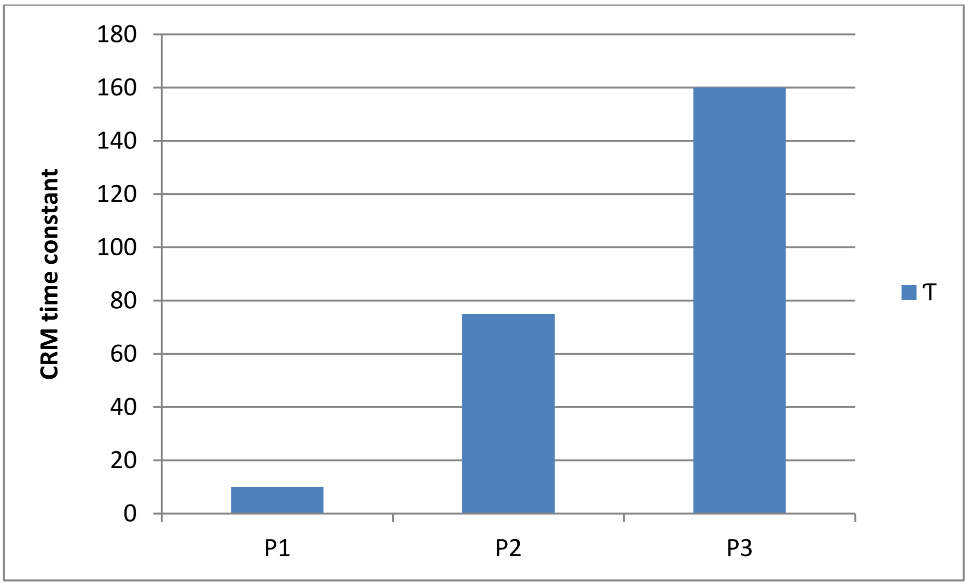

Figure 3. 22: Measured values of time constant for each producer.

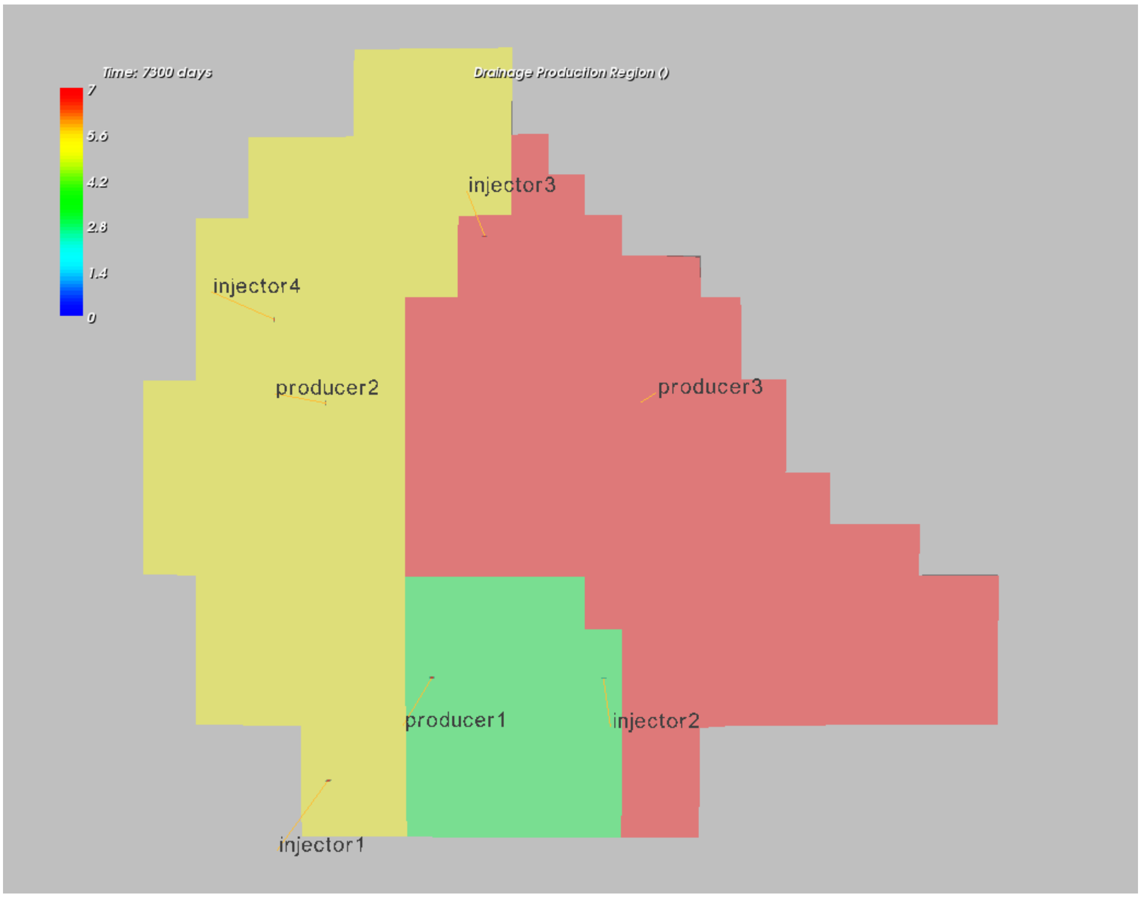

Figure 3. 23: Drainage volume of each producer calculated from reservoir simulator.

The estimated values of the liquid production rate from *CRMP* versus the values from simulation are shown in Figures 3.25 to 3.27. As can be seen, there is an acceptable match between estimated production and production history.

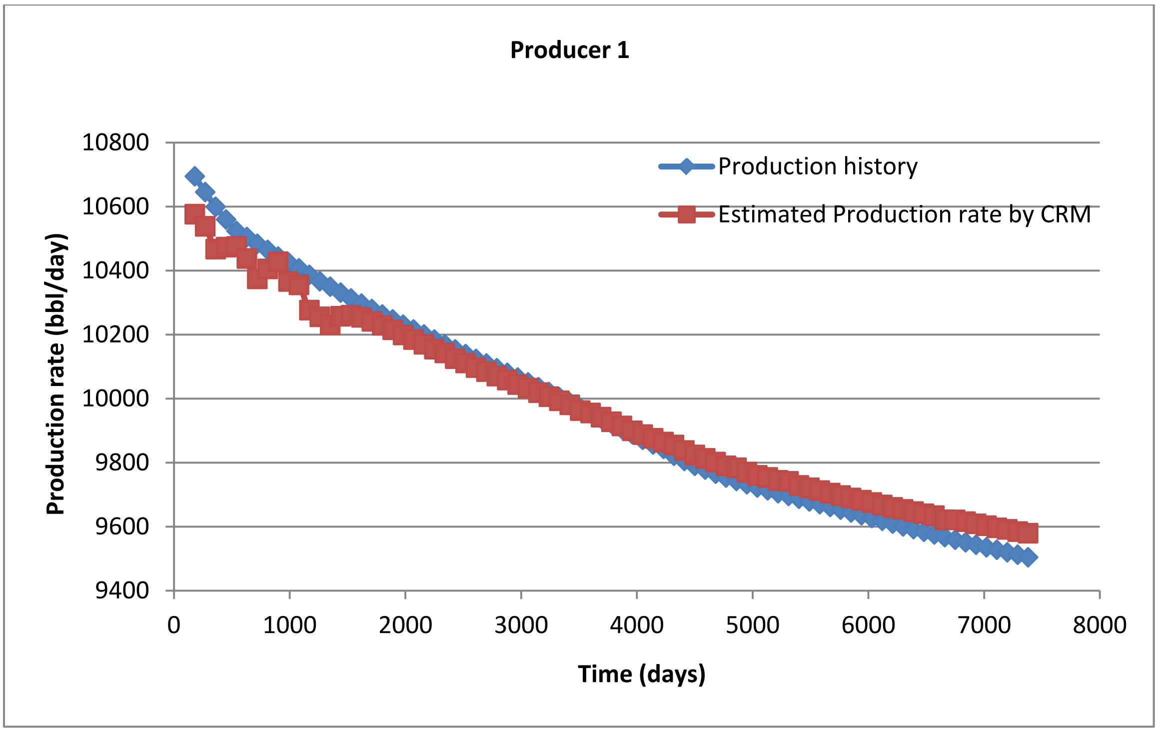

Figure 3. 24: Calculated production rate from CRMP versus production history of Producer 1.

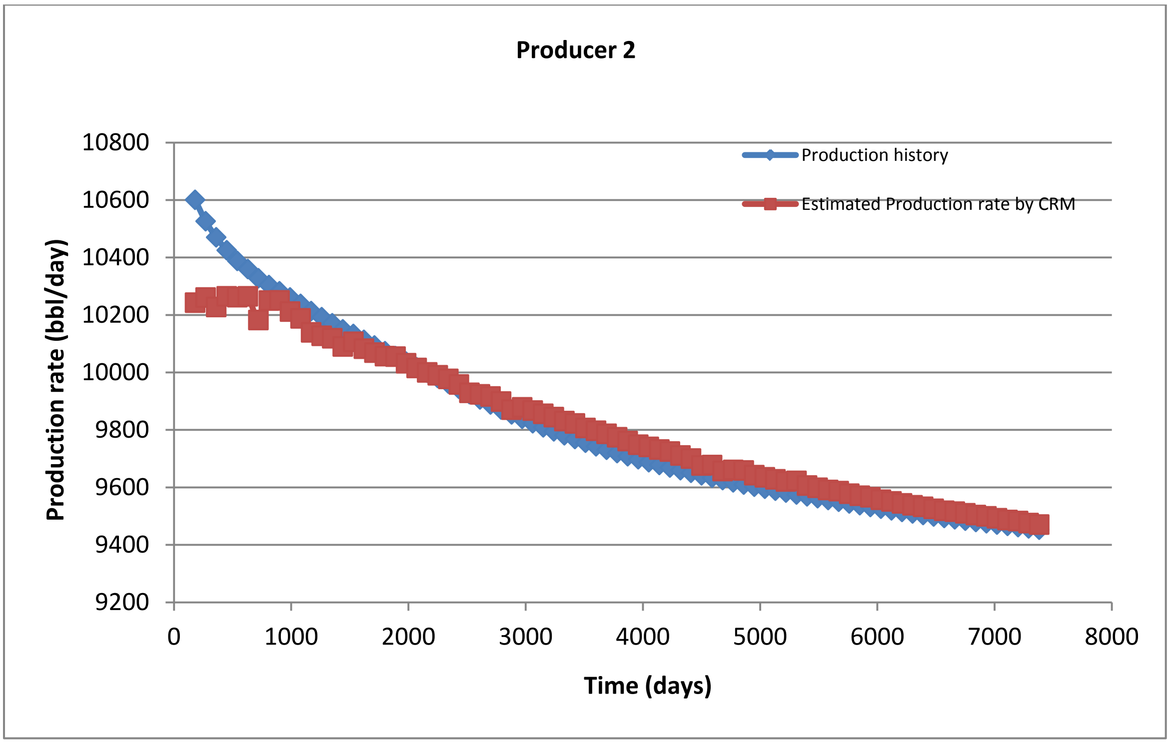

Figure 3. 25: Calculated production rate from CRMP versus production history of Producer 2.

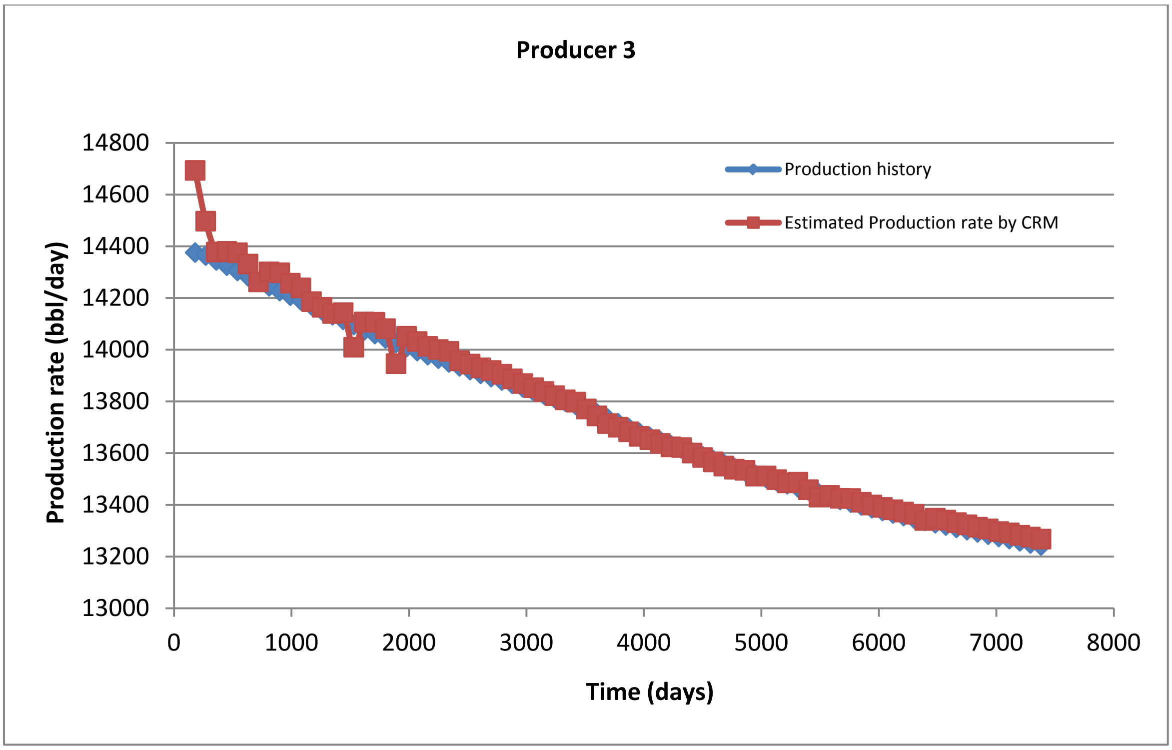

Figure 3. 26: Calculated production rate from CRMP versus results production history of Producer 3.

3.4 Allocation management

In this section a simple new approach is introduced, in which the calculated inter-well connectivity results are used to determine the allocation factor for each injector. The aim is to allocate more water to those injectors that are supporting the better producers i.e. production wells with the highest oil production performance. This requires evaluating the performance of each producer and combining the results of the connectivity measurement in order to optimise the distribution of water between the injectors. So in this new methodology, connectivity results have been coupled with the parameters describing the production well performance, to manage the water allocation.

We will start with one of the most common parameters in petroleum engineering for describing the performance of the production wells, which is the water cut (*WC*). However, *WC* should be considered in conjugation with the liquid production ratio (*LR)* of the producer (Equation 3.20) to compare the performance of each individual producer with other production wells.

$$LR = \frac{q_{lj}}{q_{lf}} \qquad \text{(Equation 3.20)}$$

in which q_{lj} is the liquid production of the well and q_{lf} is the total liquid production of the reservoir. Figure 3.28 shows the WC of each producer after 20 years of production and Figure 3.29 will shows the LR of each producer.

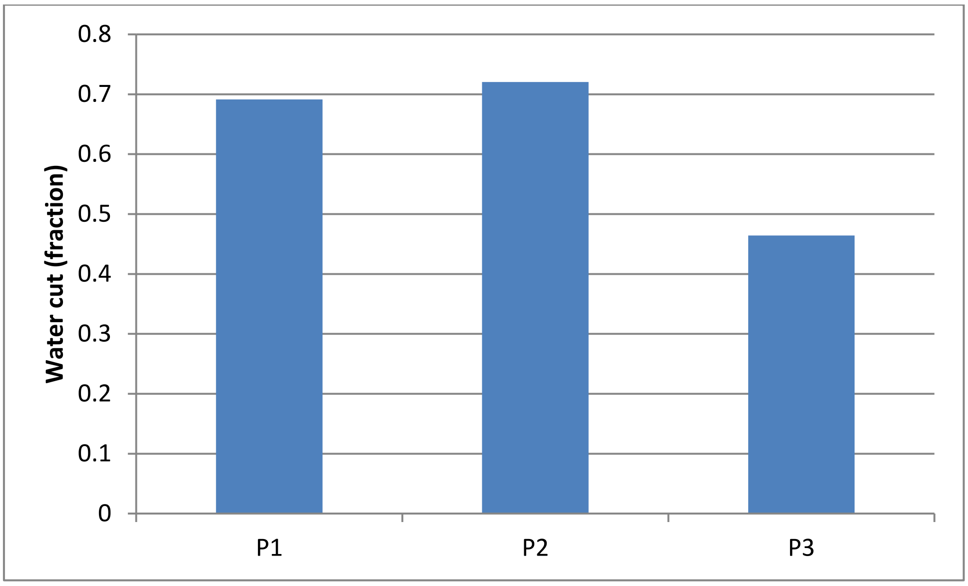

Figure 3. 27: WC of each producer at the end of 20 years of production.

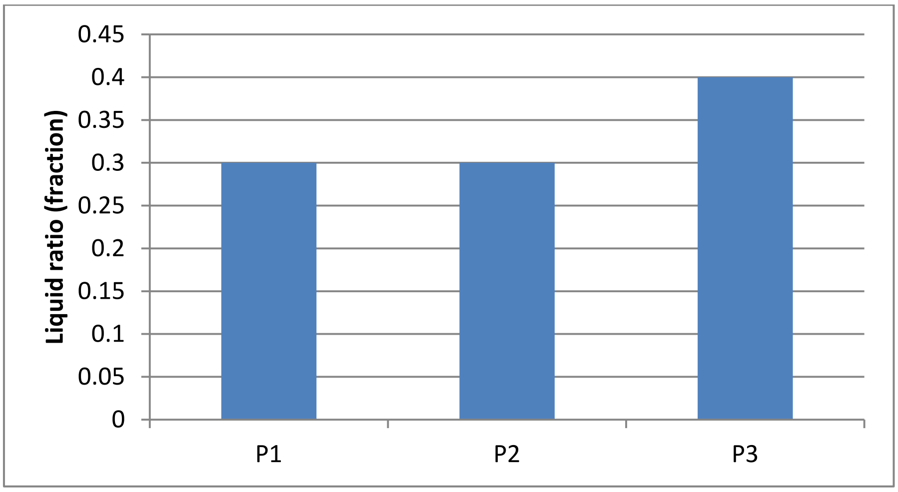

Figure 3. 28: Calculated LR of each producer at the end of 20 years of production.

Taking *LR* into account, the oil production performance index of each producer *(OI)* can be defined as:

$$OI = (1 - WC) \times LR \qquad \text{(Equation 3.21)}$$

The calculated *OI* for each producer is shown in Figure 3.29.

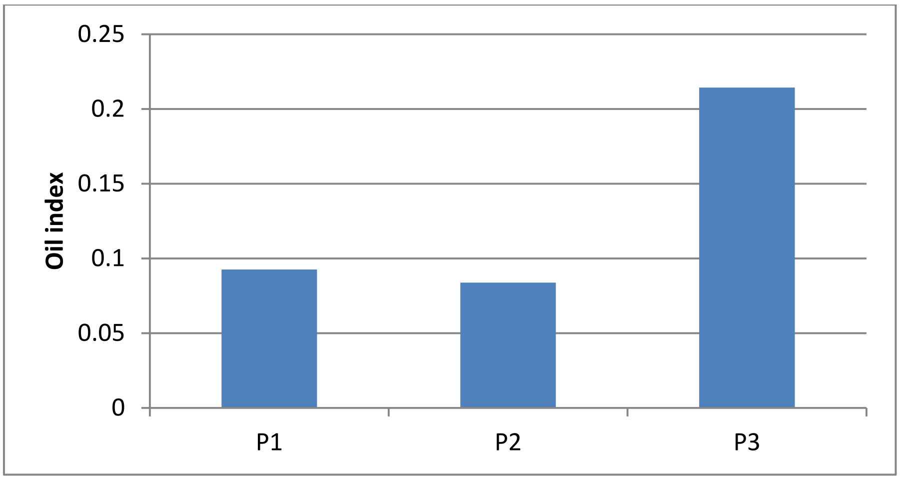

Figure 3. 29: Calculated OI of each producer at the end of 20 years of production.

OI is the parameter that describes the performance of each producer. A higher *OI* means a better oil producer. In this example, Producer 3 has the best *OI*, while Producer 2 has the lowest one. For each injector, the total water allocation index (AF_t) is defined as:

$$AF_{ti} = \sum_{j=i}^{P} (OI_j \times C_{ij}) \quad (i=1,2,\ldots,I) \qquad \text{(Equation 3.22)}$$

In the above equation AF_{ti} is the total water allocation index for injector *i*. OI_j is the oil production performance index of producer *j* and C_{ij} is the calculated inter-well connectivity from previous sections for a paired producer *j* and injector *i*. The final water allocation factor *(AF)* for each injector will be:

$$AF_i = \frac{AF_{ti}}{\sum_{i=i}^{I} AF_{ti}} \qquad \text{(Equation 3.24)}$$

The calculated *AF*s for each injector, based on the determined connectivity measurement from different techniques, are shown in Figure 3.30.

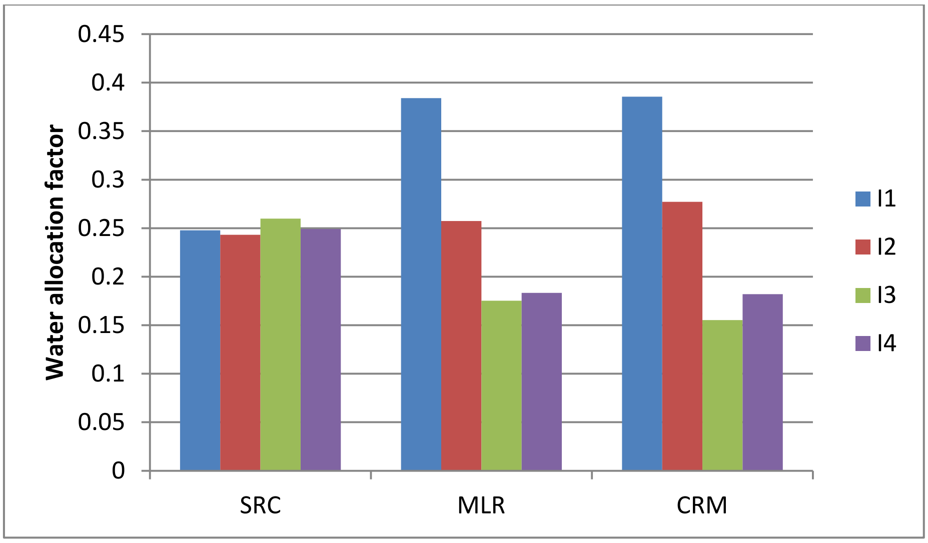

Figure 3. 30: Determined allocation factor for each injection well from different statistical methods.

3.5 Results and discussion

In the final stage of this chapter, the calculated *AF*s are used to manage the allocation of water between injectors for the next 20 years of production. The following figures show the results of each technique.

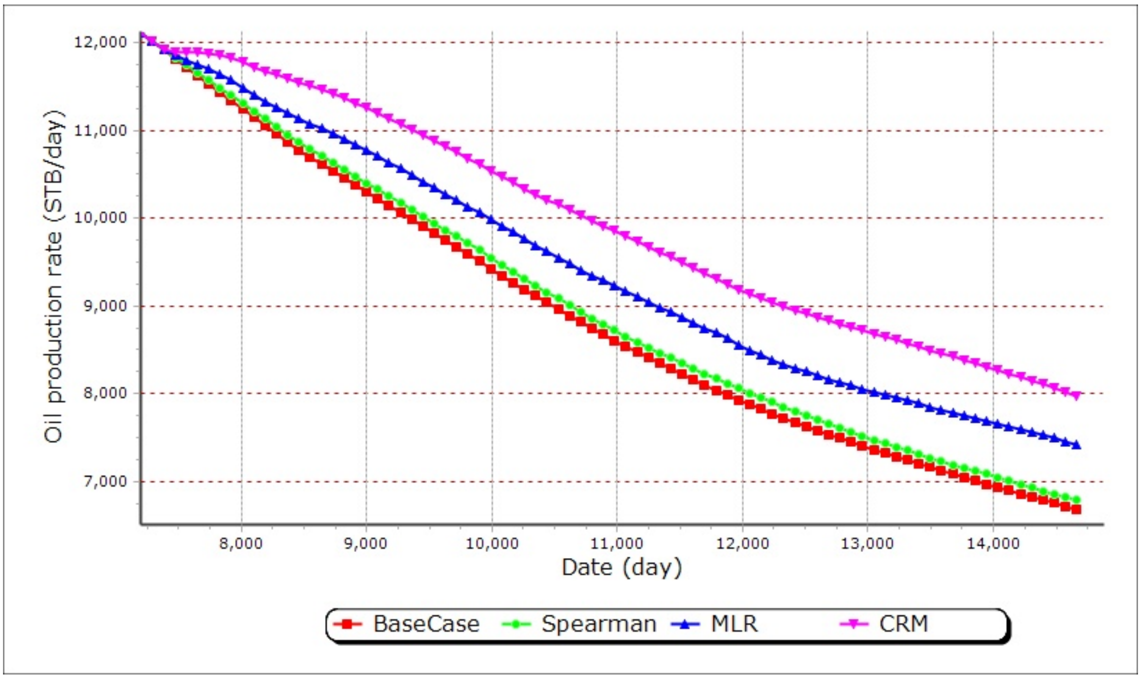

Figure 3. 31: Comparison of improvement in oil production rate after WAM based on different statistical techniques.

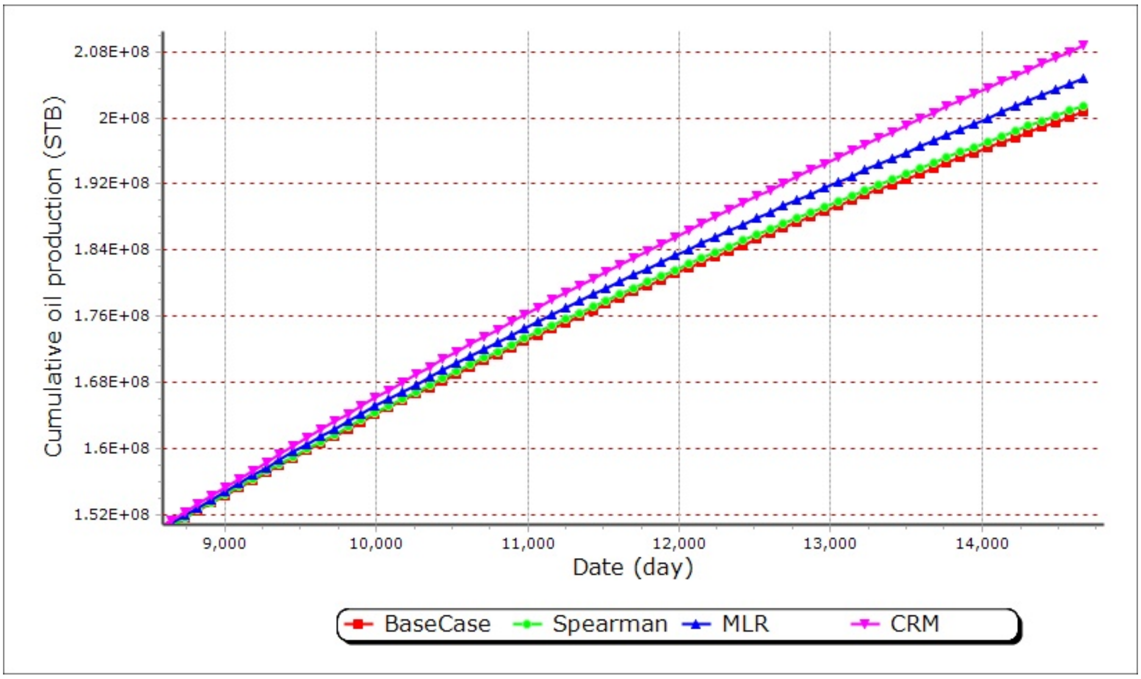

Figure 3. 32: Comparison of improvement in cumulative oil production after WAM based on different statistical techniques.

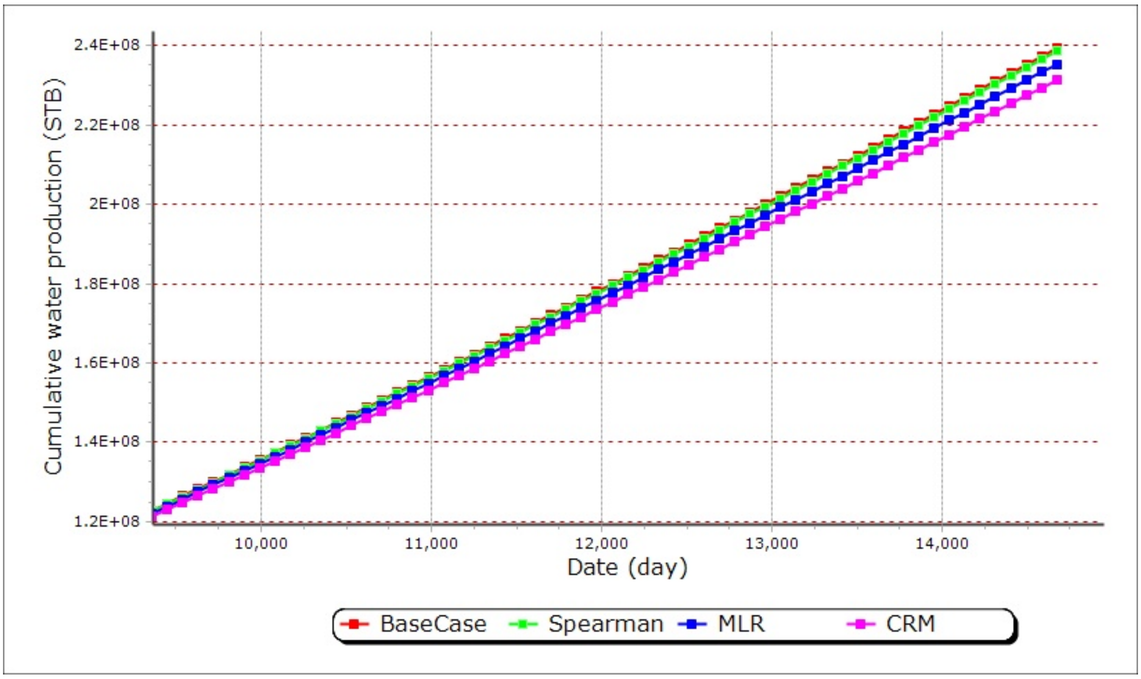

Figure 3. 33: Comparison of reduction in cumulative water production after WAM based on different statistical techniques.

The results of simulation after 40 years show improvement in waterflood performance after water allocation management. A better daily oil production profile was achieved (Figure 3.31), more cumulative oil produced (Figure 3.32) and water production decreased (Figure 3.33). All these were obtained while the same amount of water was injected into the reservoir. This means that water allocation management based on inter-well connectivity measurement and production well performance evaluation will improve the sweep efficiency in a water injection project.

However, only a very small improvement was obtained from the Spearman rank correlation. As discussed earlier in this chapter, in section 3.3.1, this technique cannot properly predict the connection between individual injectors and producers. It will only give an idea of how the total injection system is working. For example the highest values of this coefficient are related to Producer 2 (Figures 3.8 and 9), the production well that is producing more water than the others.

The results of *MLR* and *CRM* were almost the same. The proposed technique for finding the injectors connected to the producer was effective and significant improvement was achieved in overall performance of water injection. *CRM* works a little better, as it has a time constant that will take into account the lag time between producers and injectors.

Although a new technique has been developed for water allocation, it needs to be pointed out that, in reality, there are other parameters that put constraints on the allocated injection rate. For example the injection rate should match the outflow performance of the injection well, compressor and pump capacity and in most cases the injection pressure should be less than the matrix fracture pressure.

OI was a good parameter to describe the performance of the producer but we are still interested in finding another parameter that can differentiate more clearly between a good and bad producer. In addition, it should be a good representative of the production history. This will be briefly discussed in the Chapter 5.

3.6 References

1. Lee, K.-H., et al., *A Multivariate Autoregressive Model for Characterizing Producer-producer Relationships in Waterfloods from Injection/Production Rate Fluctuations*, in *SPE Western Regional Meeting*. 2010, Society of Petroleum Engineers: Anaheim, California, USA.

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Chapter 4– Producer-Injector Inter-well Connectivity Measurement; Artificial Intelligence Techniques

Neural networks, non-algorithmic, non-digital, intensely parallel and distributive information processing systems, are being used more and more every day. The main interest in neural networks is rooted in the recognition that the human brain processes information in a different manner to conventional digital computers. Artificial neural networks (*NNs*) are specialized techniques that generate strategies to map input to output data. Artificial neural networks are routinely used in complex time series prediction. A typical *ANN* involves processing patterns that evolve over time [1].

In this chapter, first a brief discussion of the artificial neural networks is presented followed by strategies to design a network for fluid flow simulation and prediction of well interaction in heterogeneous permeable media. The results are then presented and conclusions drawn based on these results.

4.1 Artificial neural networks (*ANNs*)

Artificial Neural Networks (*ANNs*) are computational modelling tools that have recently emerged and found extensive acceptance in many disciplines for modelling complex real-world problems [2]. However, if a problem is solvable by conventional methods, neural networks (or any other virtual-intelligence technique) should not be used to solve it. Although there is academic value to solving simple problems, such as polynomials and differential equations, with neural networks to show their capabilities, they should be used mainly to solve problems that otherwise are very time-consuming or simply impossible to solve by conventional methods [3].

ANNs may be defined as structures comprised of densely interconnected adaptive simple processing elements (called artificial neurons or nodes) that are capable of performing massively parallel computations for data processing and knowledge representation [4]. These networks are physical cellular systems that can acquire, store, and use experiential knowledge. The knowledge is in the form of stable states or mapping embedded in networks that can be recalled in response to the presentation of cues [5].

Much of the interest in neural networks arises from their ability to discover the underlying system by developing a function between input and output vectors on the basis of historical data. Neural networks accumulate the knowledge implicitly in connection weights between the layers. As a consequence, the knowledge can be modified by changing the weights through back-propagation. With the back-propagation rule, also referred as delta learning rule, the network first uses the input vector to produce an output, and compares this output to the desired output. In the case there is a difference, the weights are modified between the layers to further decrease the difference. This continues until the minimum desired error rate is obtained between the network produced and actual output [6].

4.1.1 Structure of ANNs

Artificial neural networks are information-processing systems that are a simplified simulation of human biological process and have the same performance characteristics as those of biological neural networks inside human body. The development of *ANNs* as generalizations of mathematical models of human cognition or neural biology is based on the these assumptions [3].

1. Information processing occurs in many neurons or simple elements that are sometimes called processing elements (PE) (Figure 4.1) [3].
2. Signals are passed between neurons over connecting links [3].
3. Each connecting link has an associated weight, which, in a typical neural network, multiplies the signal being transmitted [3].
4. The output signal of each neuron is determined by an activation function (usually nonlinear) which applied to its net input [3].

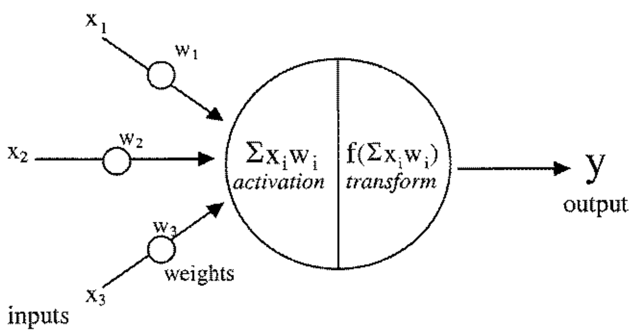

Figure 4. 1: Structural model of an artificial neuron[7].

4.1.2 Classification of ANNs

ANNs can be classified in many different ways, according to one their relevant features. In general, classification of *ANNs* may be based on (i) the function of the *ANN* (e.g., pattern association, clustering), (ii) the degree (partial/full) of connectivity of the neurons in the network, (iii) flow direction of information within the network (recurrent and non-recurrent), with recurrent networks, which are dynamic systems in which the state at any given time is dependent on previous states, (iv) the learning algorithm type, which represents a set of systematic equations that employ the outputs obtained from the network along with an arbitrary performance measure to update the internal structure of the *ANN*, (v) the learning rule (the driving engine of the learning algorithm), and (vi) the degree of learning supervision needed for training the *ANN*. Supervised learning will train an *ANN* when the correct answers (i.e. target outputs) are provided for every example, and the solution of the *ANN* is compared to the corresponding target values to determine the required amount by which each weight should be adjusted. Reinforcement learning is supervised; however, the *ANN* is provided with a critique on correctness of output rather than the correct answer itself. Unsupervised learning does not require a correct answer for training, however the network, through exploring the underlying structure in the data and the correlation between the various examples, organizes the examples into clusters (categories) based on their similarity or dissimilarity. Finally, the hybrid learning procedure combines supervised and unsupervised learning [2].

4.1.3 Application of ANN

ANNs have been utilized in a variety of applications, such as modelling, classification, pattern recognition, and multivariate data analysis.

4.1.3.1 Pattern classification

Pattern classification will use supervised learning to assign an unknown input pattern, to one of several pre-specified classes based on one or more properties that characterize a given class, as shown in Figure 4.2 [2].

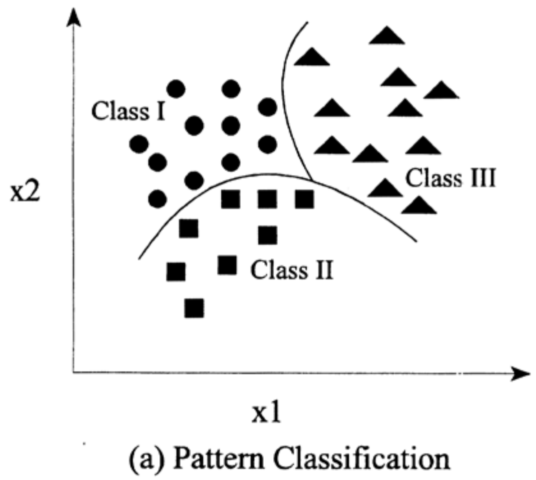

Figure 4. 2: An example of pattern classification by ANN [2].

4.1.3.2 Clustering

Clustering can be performed by unsupervised learning in which the clusters (classes) are formed by exploring the similarities or dissimilarities between the input patterns according to their inter-correlations (Figure 4.3). The network will assign 'similar' patterns to the same cluster [2].

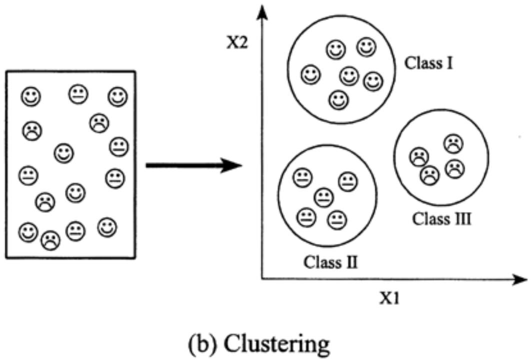

Figure 4. 3: An example of clustering by ANN[2].

4.1.3.3 Function approximation (modelling)

Function approximation (modelling) concerns training the *ANN* on input-output data in order to explore the underlying rules relating the inputs to the outputs (Figure 4.4). Multilayer *ANNs* are considered as universal estimators that can approximate any arbitrary function to any degree of accuracy [8], and therefore are normally employed in this application. This type of *ANN* is applied to problems (i) where there is no theoretical model, i.e. data obtained from experiments or observations are utilized, or (ii) to substitute theoretical models that are hard to compute analytically by utilizing data obtained from such models [2].

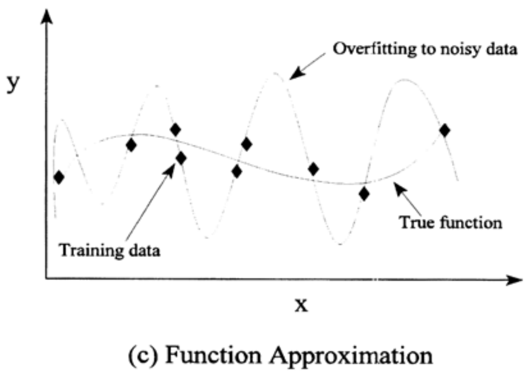

Figure 4. 4: An example of function approximation by ANN[2].

4.1.3.4 Forecasting

Forecasting includes training of an *ANN* on samples from a time series representing a certain phenomenon at a given scenario and then using it for other scenarios to predict (forecast) the behaviour at subsequent times (Figure 4.5). That is, the network will predict Y(t + 1) from one or more previously known historical observations [e.g., Y(t - 2), Y(t - 1), and Y(t), where *t* is the time step [2].

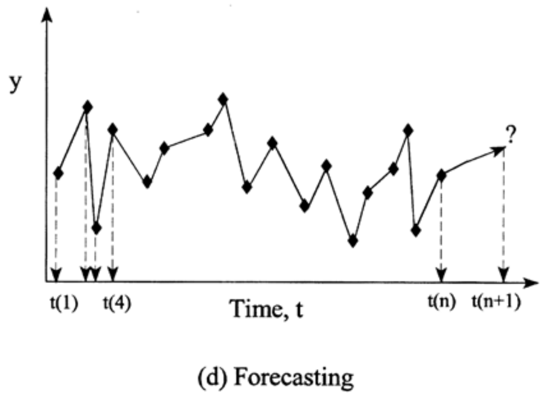

Figure 4. 5: An example of forecasting by ANN [2].

4.1.4 Application of ANN in petroleum engineering

Applications of *ANNs* in petroleum engineering can be divided into two categories: those that use neural networks to analyse formation lithology from well logs and those that use neural networks to pick a reservoir model to be used in conventional well test interpretation studies. These tasks are usually done by log analysts and reservoir engineers, and their automation using a fault-tolerant process may prove valuable[5].

Neural networks can help engineers and researchers by addressing some fundamental petroleum engineering problems as well as specific ones that conventional computing has been unable to solve. Petroleum engineers may benefit from neural networks on occasions when engineering data for design and interpretations are less than adequate [5].

Neural networks have proved to be valuable pattern-recognition tools. They are capable of finding highly complex patterns within large amounts of data. A relevant example is well log interpretation. It is generally accepted that there is more information embedded in well logs than meets the eye. Thus, determination, prediction, or estimation of formation permeability without actual laboratory measurement of the cores or interruption in production for well test data collection has been a fundamental problem for petroleum engineers [5].

Neural networks have shown great potential for generating accurate analysis and results from large historical databases, the kind of data that engineers may not consider valuable or relevant in conventional modelling and analysis processes. Neural networks should be used in cases where mathematical modelling is not a practical option. This may be because all the parameters involved in a particular process are not known and/or the interrelation of the parameters is too complicated for mathematical modelling of the system. In such cases, a neural network can be constructed to observe the system behaviour (what type of output is produced as a result of certain set of inputs) and try to mimic its functionality and behaviour [3].

4.2 *ANN* and inter-well connectivity measurement

Consider a pair of injection-production wells in a homogeneous cross section where q_w is the injection rate of water and q_l is the liquid production rate. For this simple system we can express q_l as:

$$q_l = Z(q_w) \qquad \text{(Equation 4.1)}$$

where Z is the transfer function which models the fractional flow characteristics of the medium. For a simple system, as in Equation 4.1, an *ANN* can easily be designed by using the injection history as the input data to the network and production rates as the output. [1] Therefore, an *ANN* can be designed to forecast the liquid production of a production well by inputting the injection rate of the surrounding injectors. By sequentially varying the injection rates of the injectors around a target well one can determine the relative influence of each of the injectors surrounding a target well [1].

In this section we will give a short introduction to two types of *ANNs* used in this research.

4.2.1 Feed-forward back propagation (FFBP)

Feed-forward networks are a generalization of the multi-layer perceptron (*MLP)*, such that connections can jump over one or more layers. In theory, an *MLP* can solve any problem that a generalized feed-forward network can solve. In practice, however, generalized feed-forward networks often solve the problem much more efficiently. A classic example of this is the two spiral problem. A standard *MLP* requires several hundred times more training steps than a generalized feed-forward network with the same number of processing elements.

4.2.2 Fuzzy logic network (Co-adoptive neuro-fuzzy inference system CANFIS)

The *CANFIS* (Co-Active Neuro-Fuzzy Inference System) model integrates adaptable fuzzy inputs with a modular neural network to rapidly and accurately approximate complex functions [9]. Fuzzy inference systems are also valuable as they combine the explanatory nature of rules (membership functions) with the power of "black box" neural networks.

4.3 Case Study

The same reservoir model as that used in Chapter 2 (Section 2.2.1) has been employed in this analysis.

As previously described, inter-well connectivity is determined based on 20 years of injection and production rate history (Figures 3.6 and 7). The injection water is then re-allocated to the injectors for the next 20 years of production, based on the results of the connectivity calculation.

Both types of *ANNs* represented above (*FFBP* and *CANFIS*) were employed.

The statistical techniques described in Chapter 3 allowed direct calculation of inter-well connectivity coefficients. However, these coefficients cannot be determined directly when employing *ANNs*. This difficulty was overcome by:

1. An *ANN* is designed for each producer in order to estimate the rate of liquid production based on the injection rates of the connected injectors.

2. The designed *ANN* is used to run a sensitivity analysis to monitor the effect of the change in the injection rates of the connected injectors on the production rate of the producer. The results of this sensitivity analysis will show the inter-well connectivity between the producer and its surrounding injectors.
3. At the end, an allocation factor will be determined for each injector, based on its connection to the producers and the producing water cut of the production wells

4.3.1 Feed-forward back propagation (FFBP)

The process employed in the application of *FFBP* will be described in detail since it is the first *ANN* technique being employed.

4.3.1.1 Designing the optimum network

The main steps in designing and developing an efficient neural network for each producer are:

1. Defining the base design
2. Preparing a data set for training the base case
3. Carrying out super-position analysis to identify which injector is connected to each of the production wells.

A. Base case design

The initial base case is designed based on the number of inputs and outputs and the volume of data available for each parameter. The network has one output, the liquid production rate of the producer. The number of inputs depends on how many injectors are connected to each producer. For the base case, it has been assumed that the producer is only supported by the nearest two injectors. The properties of the base case network for each producer are given in Table 4. 1.

Table 4. 1 Initial properties of the FFBP network designed to estimate liquid production rate of producer number one.

Number of Hidden layers	1
Number of processing elements (PE)	4
Hidden layer transform function	Sigmoid
Output layer transform function	Sigmoid
Number of epochs for training	1000

The transfer (activation) function [10] is necessary to transform the weighted sum of all signals impinging onto a neuron so as to determine its firing intensity. Most *ANNs* utilizing back propagation (*BP*) employ a sigmoid function, which possesses the distinctive properties of continuity and differentiability, essential requirements in *BP* learning [2].

The number of neurons in the hidden layers is very important, since it affects the training time and generalization property neural networks. On the one hand, too many neurons may cause the network to memorize (over-fitting) as opposed to generalize; on the other hand too few neurons would require more training time in finding the optimal representation or generally result in under-fitting. We adjusted the number of neurons in the hidden layer experimentally One rule of thumb in the neural network literature indicates that the number of neurons in a hidden layer should be 2/3 of the number of input neurons plus the number of output neurons [6].

B. Training the network

The most critical aspect of a successful *ANN* design is the selection of an input vector that is general enough for the network to train on efficiently. A poorly chosen input vector will yield an ill-formed transfer function or weight matrix that will not converge during the training or will yield inaccurate results during the prediction phase. The selection of the input vector becomes more complicated as the nature of the process under consideration becomes more complex. If the window of the input vector is sub-optimal the network will fail to generalize, which will result in poor learning and inaccurate prediction.

The following figures (Figures 4.6 to 4.8) show how different selection of data for training will affect the network performance (these figures are related to the producer 1 and injectors 1 and 3). This is a very important step in designing an *ANN*. Improper data allocation will result in an accurate or unreliable network. The training and testing data set should be selected in a way that respects all different ranges of samples in the inputs and outputs. In all these selections, 80% of the data were used for training and 20 percent are used for testing.

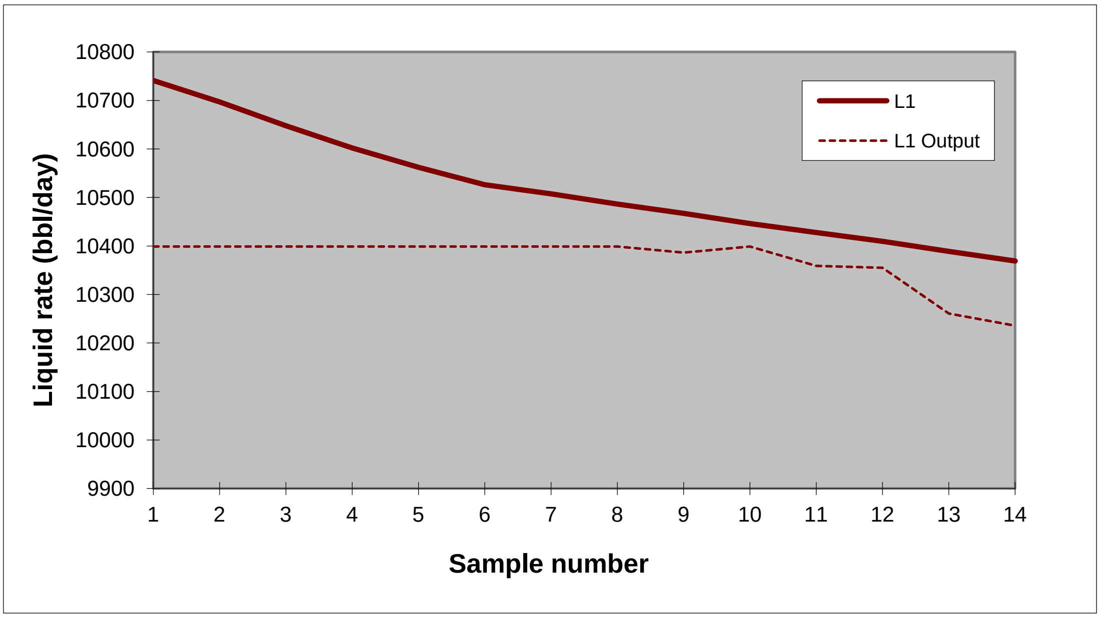

Figure 4. 6: Testing results for the trained network for producer 1 (L1 is the actual liquid rate for producer 1 from reservoir simulation and L1 output is the estimated values from the network) when the first 80% portion of the data is used for training.

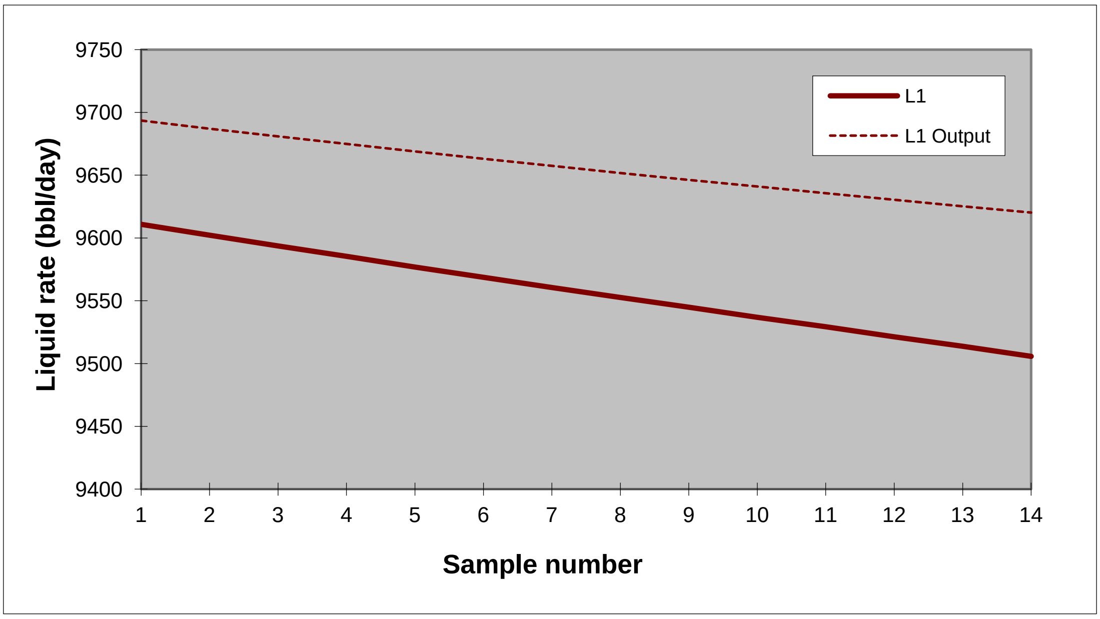

Figure 4. 7: Testing results for the trained network for producer 1 (L1 is the actual liquid rate for producer 1 from reservoir simulation and L1 output is the estimated values from the network) when the last 80% portion of data is used for training.

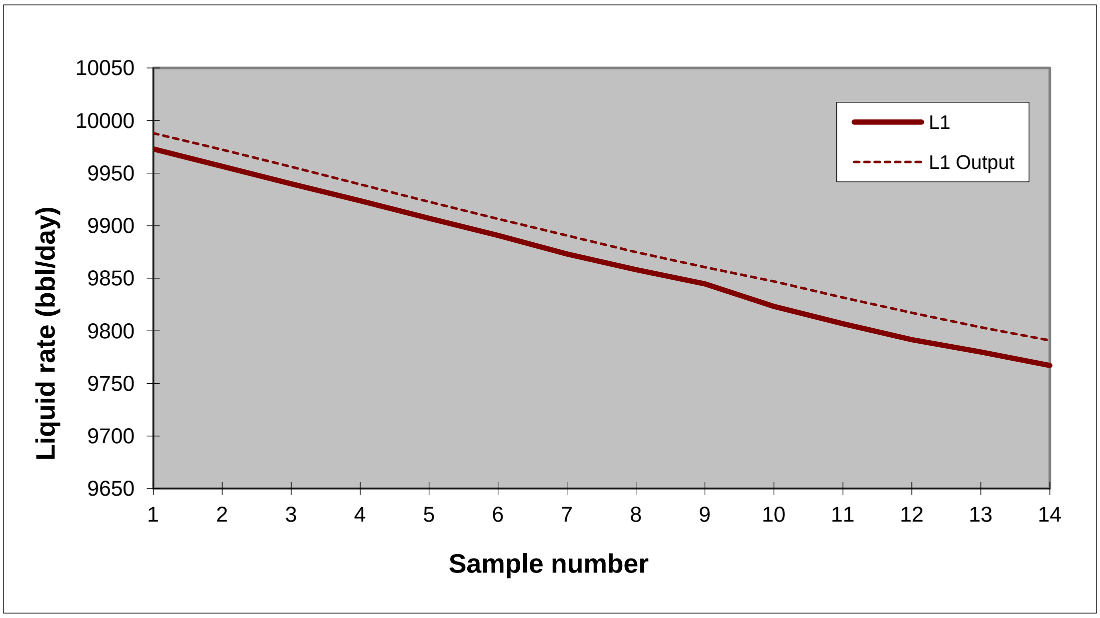

Figure 4.8: Testing results for the trained network for producer 1 (L1 is the actual liquid rate for producer 1 from reservoir simulation and L1 output is the estimated values from the network) when the first 40% portion and the last 40% portion of the data are used for training.

This analysis shows that the data selection for training and testing the network has a significant effect on the performance of the network. In order to make the network representative of all the data in this research, the histogram of the data is plotted (Figure 4.9). Then 80% percent of data in each interval are assigned to training and the rest are used for testing. This helped us to respect the whole range of the available data in the training of the network. Figure 4.10 shows the results of the actual and estimated data from the new trained network. As can be seen, there is a good agreement between them.

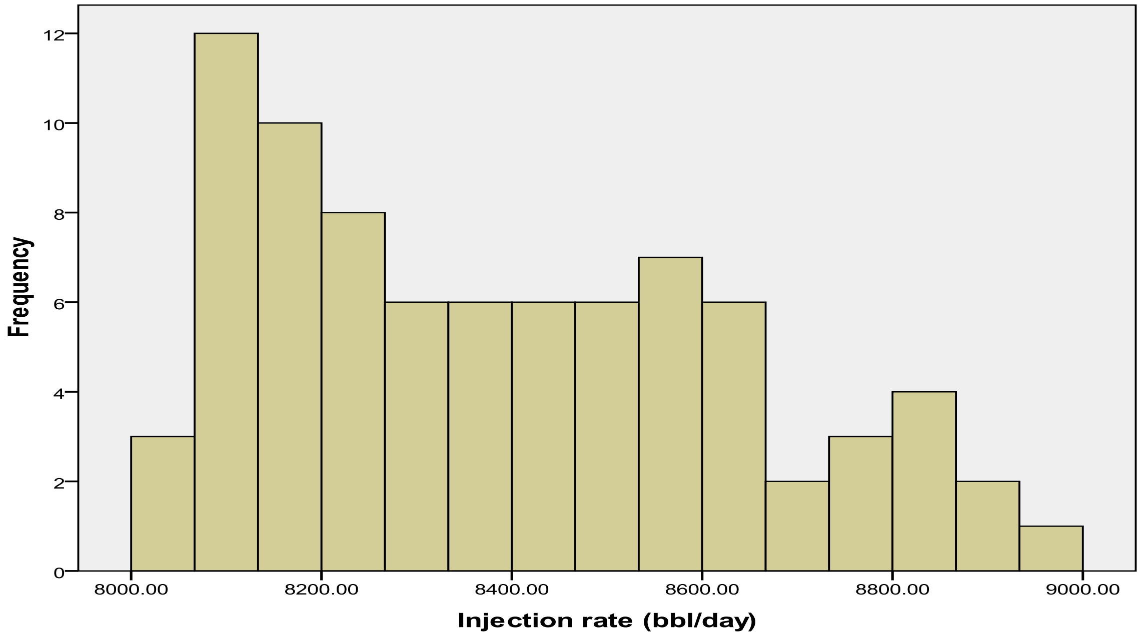

Figure 4. 9: Histogram of input data (liquid production rate) of producer 1.

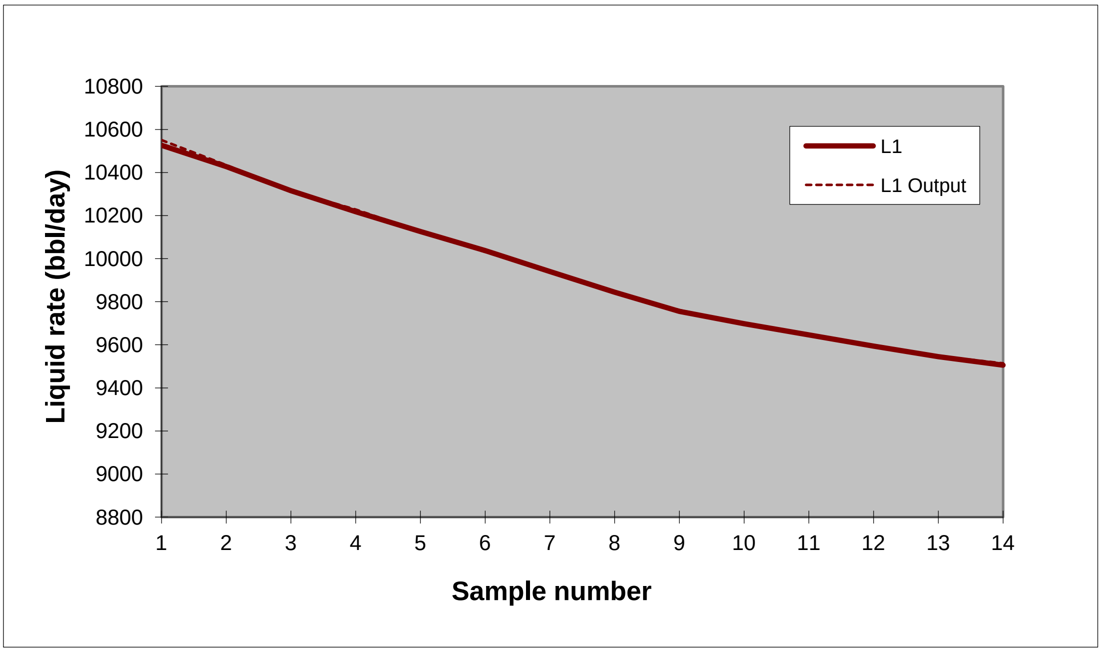

Figure 4. 10: Test results of the optimum trained network, where L1 is the actual liquid rate from reservoir simulation and L1 output is the estimated results from the network.

C. Superposition analysis

For each producer, we assume that it is connected to the two nearest injectors; this is because we do not have any information about the geology and formation structure of the reservoir, so it is more likely that the nearest wells are connected to each other. We will thus define a base case network that determines the production rate of the producer by using the injection rate of the two injectors. After that, we will try to find the optimum parameters of the network. This optimum designed *ANN* will be used to estimate the rate of liquid production. Then we add the nearest injector as a new input to the network. We will compare the error of the estimation for both cases. The third injector will be added to the network if there is decrease in the error of estimation. This process will be continued, to see which injector is connected to the selected producer. Again, since the number of inputs has changed, the number of parameters in the network should be optimized.

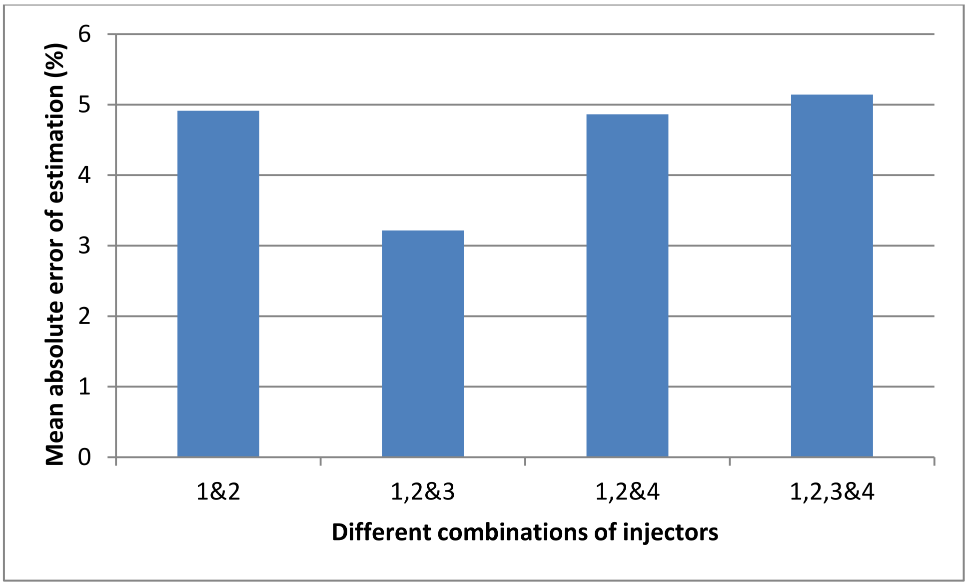

Figure 4. 11: Mean absolute error of the estimated production rate obtained from designed networks based on different combinations of injectors with producer 1.

Table 4. 2: Results of connectivity analysis for producer 1.

	Injector 1	Injector 2	Injector 3	Injector 4
Producer 1	√	√	√	×
Producer 2	×	×	√	√
Producer 3	√	√	√	×

Figure 4.11 represents the superposition analysis for producer number one: as can be seen, the best liquid production estimation came from a combination of this producer and injectors 1, 2 and 3. Table 4.2 shows the results of this analysis for all producers. Now the number of inputs for each producer's network is determined, the next step will be optimizing the network parameters.

For each parameter, we changed the number of elements then we monitored the effects of this change on the absolute error of the estimation. The optimum point is the point with the lowest error. The results of this sensitivity study for producer 1 can be seen in the Figures 4.12 to 4.14.

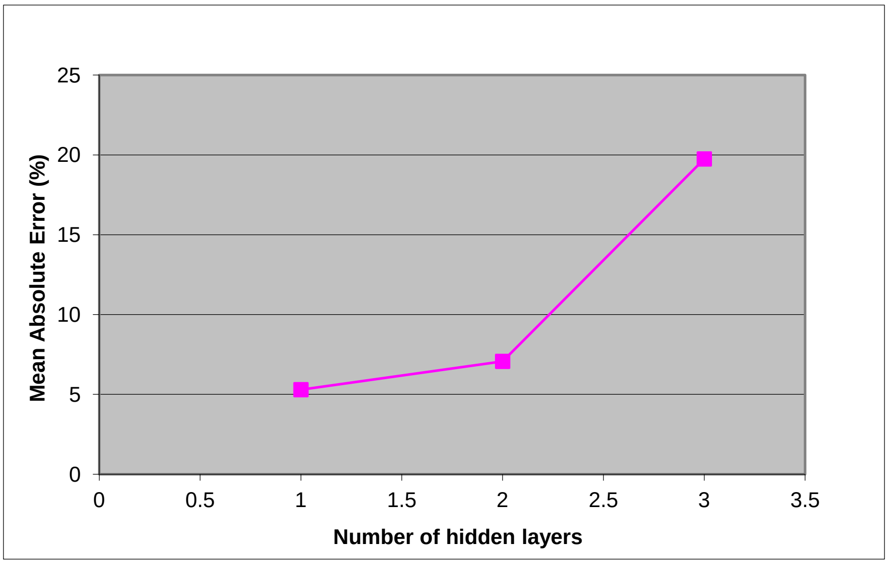

Figure 4. 12: Sensitivity analysis on the number of hidden layers shows that by increasing number of hidden layers, error of estimation will increase too.

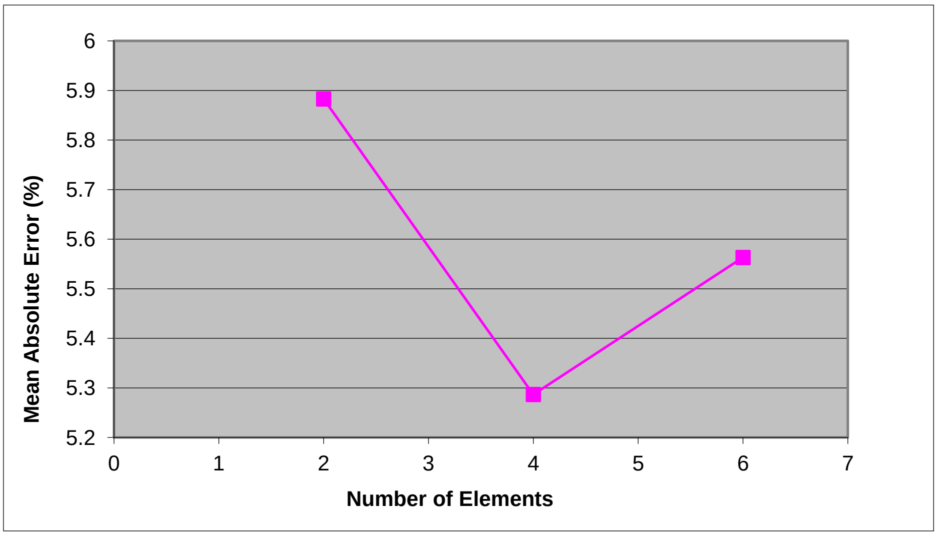

Figure 4. 13: Sensitivity analysis on the number of processing elements.

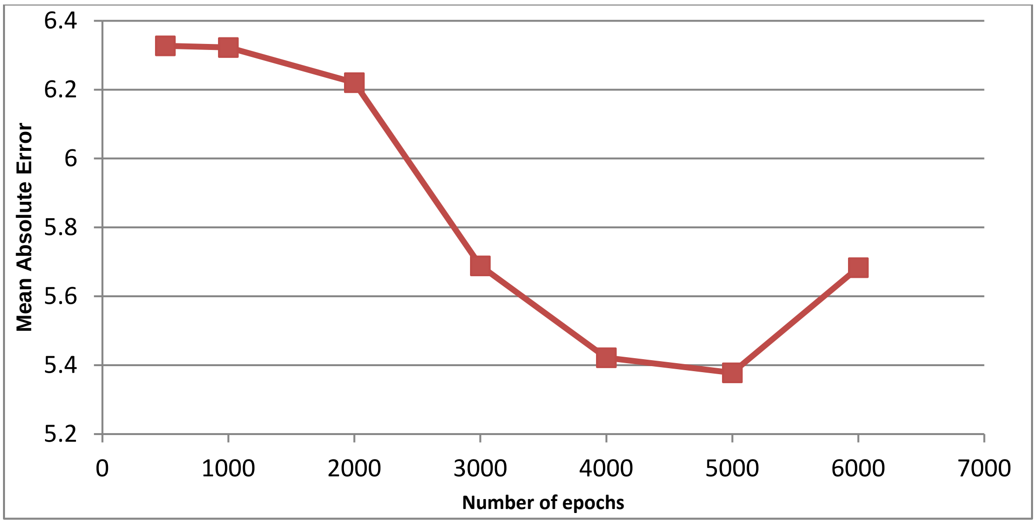

Figure 4. 14: Sensitivity analysis on the number of iterations for training the network.

Table 4.3 shows the optimum properties for the network designed for estimating the liquid production rate of the producer 1 from the injection rate of the injectors.

Table 4. 3: Optimum properties of the desired network for estimatiing liquid production rate of producer 1.

Number of hidden layers	1
Number of PE	4
Hidden layer transform function	Sigmoid
Output layer transform function	Sigmoid
Number of epochs in training	5000

The optimum network is designed to estimate the rate of liquid production based on the injection rate of the connected injector to producer 1. Figure 4.15 shows the results of the testing. This network can be used to forecast the future production rate of producer 1 by inputting the injection rate of the injectors. Also, it can be used to carry out connectivity analysis.

4.3.1.2 Inter-well connectivity analysis

To determine how an injector is supporting the producer, the injection rate of that injector is changed while the rate of other connected injectors kept constant and the rate of liquid production is calculated. Figure 4.16 shows the results of the connectivity study for producer 1. According to this figure, it can be concluded that injector 1 has the highest impact on producer 1 while injector 3 provides the lowest support.

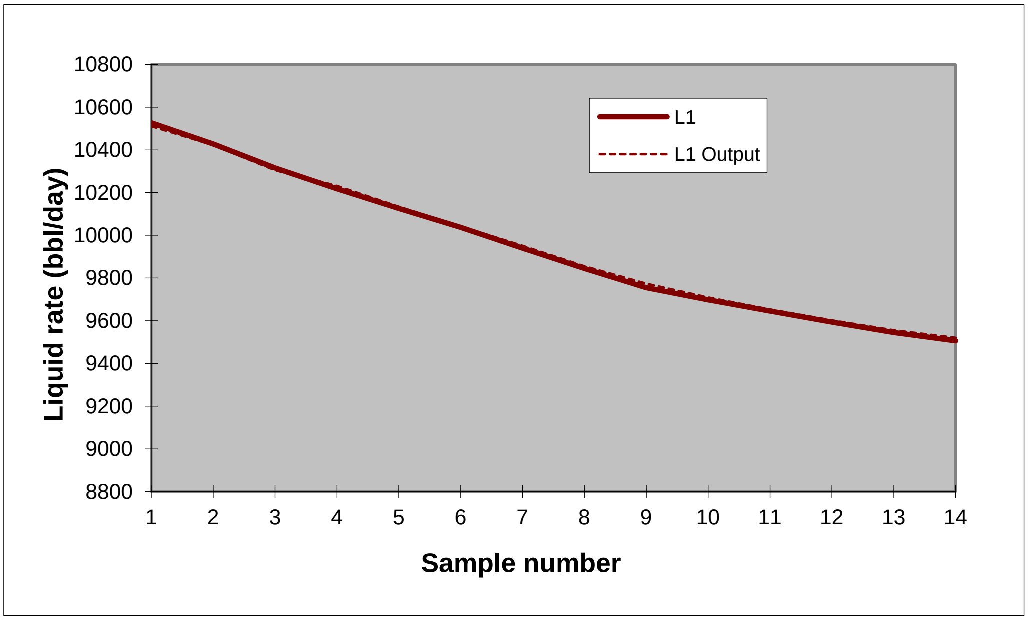

Figure 4. 15: Optimum network test results , where L1 is the actual liquid rate from reservoir simulation and L1 output is the estimated result from the network.

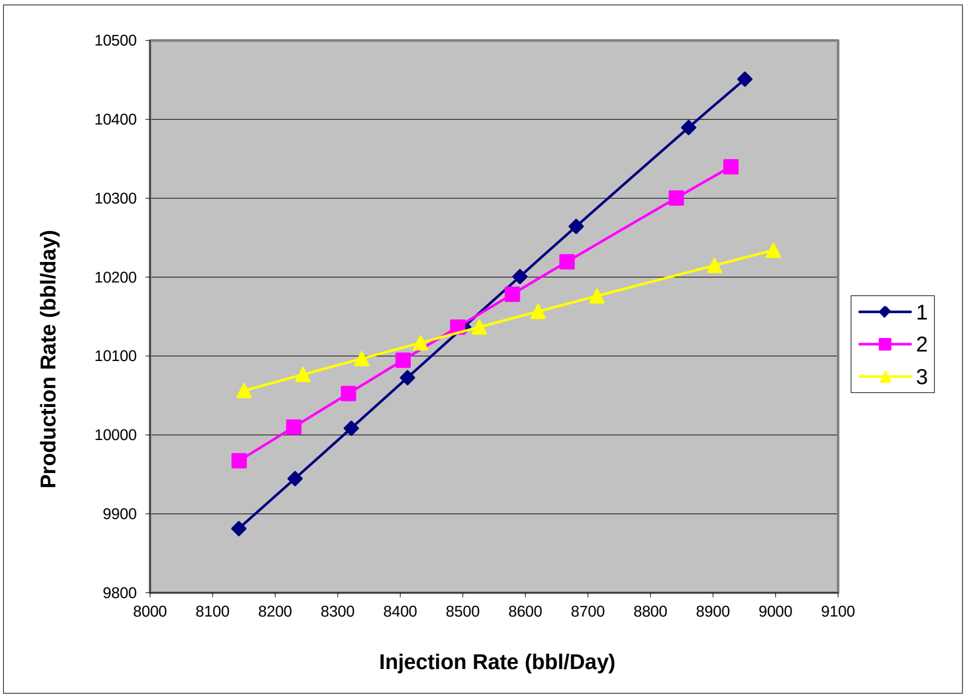

Figure 4. 16: Inter-well connectivity analysis for producer 1 (colours represent the change in liquid production of the producer by change in the rate of associated injector and numbers are the injection well numbers).

The same procedure was applied for the rest of the producers; Figures 4.17 and 4.18 represent the connectivity analysis for producers 2 and 3, and the calculated inter-well connectivity for all producers can be seen in Figure 4.19.

Figure 4. 17: Inter-well connectivity analysis for producer 2 (colours represent the change in liquid production of the producer by change in the rate of the associated injector).

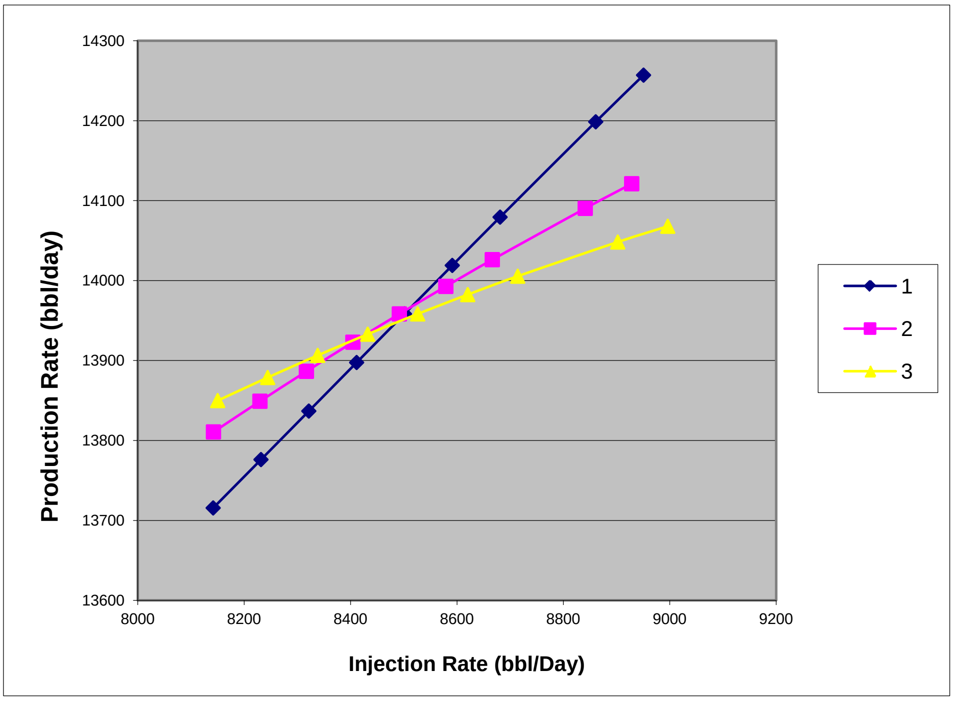

Figure 4. 18: Inter-well connectivity analysis for producer 3 (colors represent the change in liquid production of the producer by change in the rate of associated injector).

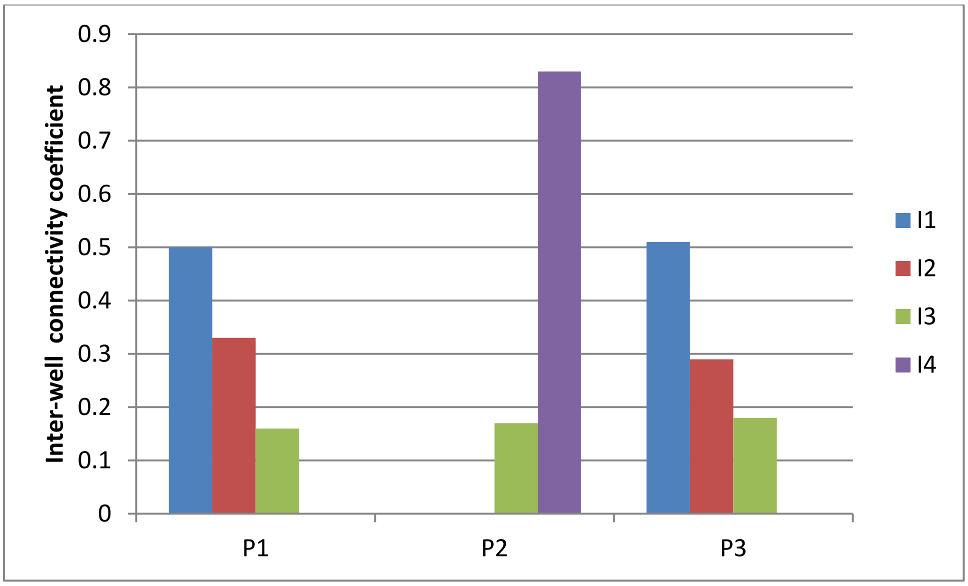

Figure 4. 19: Results of inter-well connectivity measurements for all producers by FFBP.

4.3.2 Co-Active Neuro-Fuzzy Inference System (CANFIS)

The second network that has been employed is *CANFIS*. The same procedures as have been applied for *FFBP* are employed to design the optimum network for each producer, to run the superposition analysis and to determine the inter-well connectivity. Table 4.5 will show the properties of the optimized network. Results of calculated inter-well connectivity are given in figure 4.21.

Table 4. 4: Properties of the CANFIS Network.

Transform Function	Bias Axon
Number of PE	5
Number of epochs in training	1000

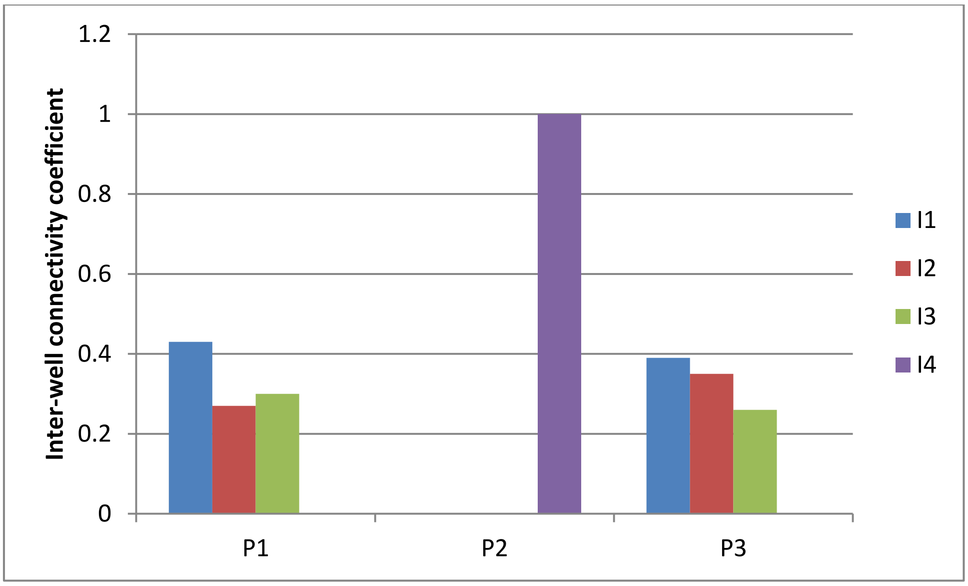

Figure 4. 20: Results of connectivity measurement for all producers from CANFIS.

4.3.3 Determination of water allocation factor

The procedure described in Chapter 3, Section 3.4, for determining the injection allocation factor was applied to the connectivity results of both networks to determine the water allocation factor .

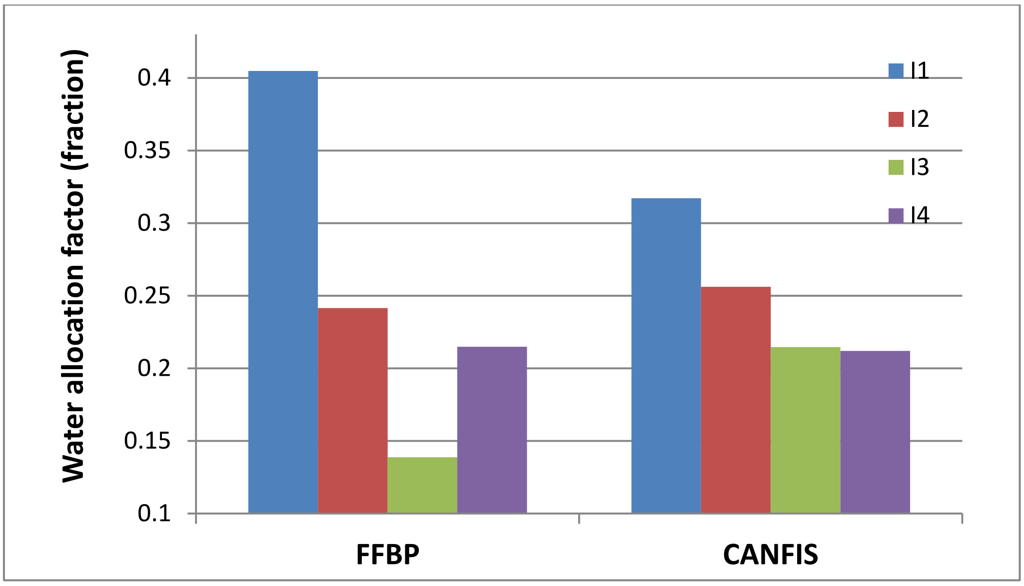

Figure 4. 21: Calculated allocation factors for both networks.

4.4 Results and discussion

In the final stage of this chapter the calculated *AF*s are used to manage the allocation of water between injectors for the next 20 years of production. The following Figures show the results of each technique.

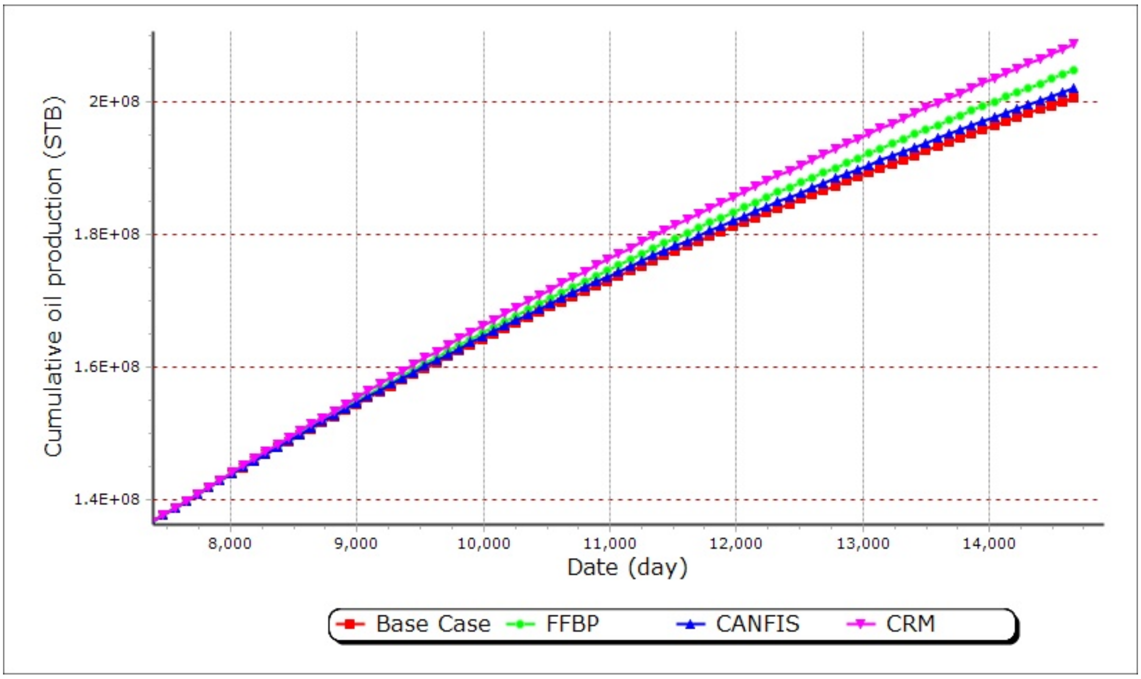

Figure 4. 22: Cumulative oil production obtained from WAM, based on FFBP, CANFIS and CRM.

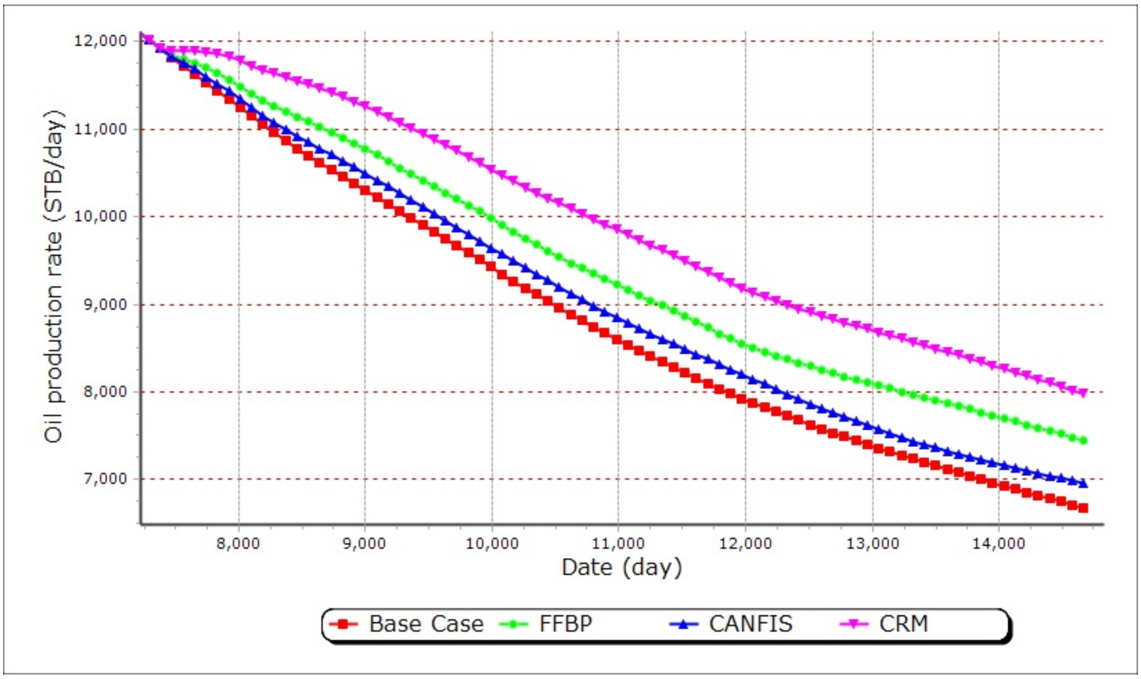

Figure 4. 23: Comparison of oil production rate from WAM, based on FFBP, CANFIS and CRM.

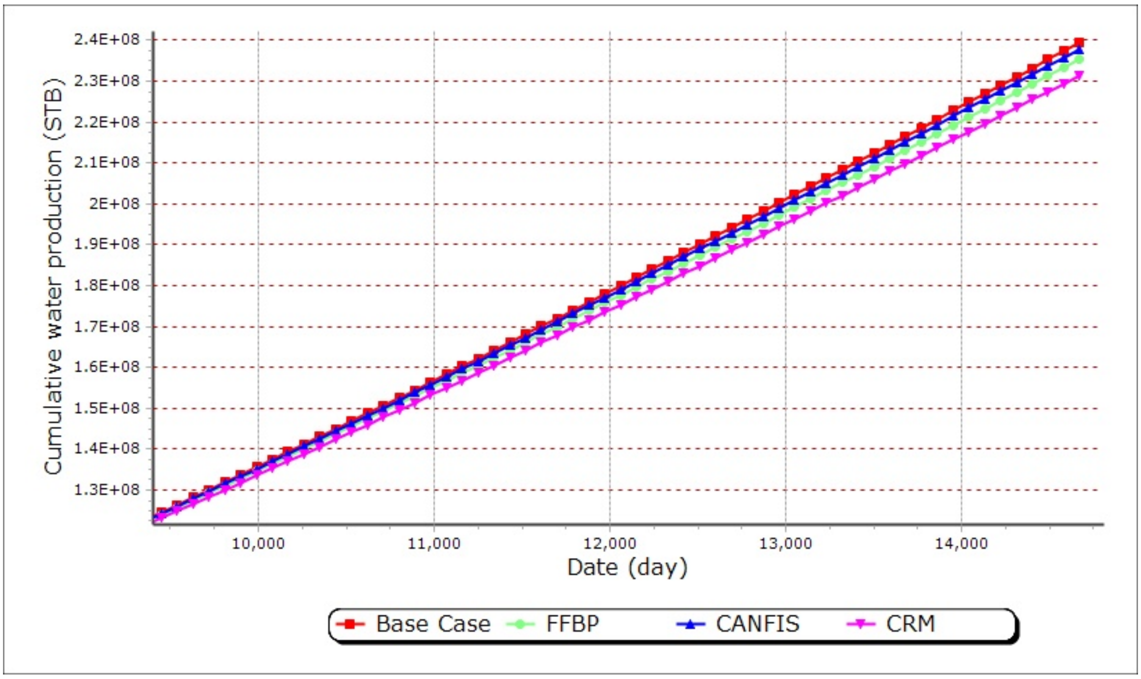

Figure 4. 24: Cumulative water production from WAM, based on FFBP, CANFIS and CRM.

As can be seen from the results of the simulation, including daily oil production rate, cumulative oil production and cumulative water production, the best results were obtained from Feed-forward back propagation. *CANFIS*-based *WAM* also improved the flooding efficiency. It seems that *BP* can work better than fuzzy logic for determining the inter-well connectivity. Comparing them with *CRM* shows that *CRM* is the winner in terms of more improvement in waterflood performance. And it indicates that *CRM* works better in determining the inter-well connectivity, as it contains more parameters related to the production and injection well connection. And also it is much easier to use *CRM* to measure connectivity. Firstly, there is no need to run a sensitivity analysis and second, it is simpler to apply. Therefore for the rest of this study we decided to work based on the connectivity results obtained from *CRM*.

4.5 References

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Chapter 5– New Allocation Management Methodologies

Although several statistical [1-7] and artificial intelligent techniques [8, 9]have been introduced to measure the connection between wells in previous studies, there has been less effort directed towards using the inter-well connectivity information for managing and improving the allocation of the water between injectors. In section 3.4 of Chapter 3 a simple new approach is introduced, in which the calculated inter-well connectivity results combined with water cut to determine the allocation factor for each injector.

Water allocation management aims to inject the water in a manner that increases the total oil recovery for a given volume of water. The “good” injectors are thus those which support the “good” producers. The previous two chapters used the water cut as a parameter for describing the performance of the producers. In this chapter I will try to find a better way to quantify a production well’s performance. Therefore further new techniques are developed: first *WC* methodology is extended to cumulative water cut (*CWC*) and then new parameters are defined for better description of production well performance. New procedures are then defined for water allocation management, based on these new parameters and inter-well connectivity measurements.

The reservoir model used in Chapters 3 and 4 will be employed again, with the inter-well connectivity measurements obtained from the *CRM*.

5.1 Water allocation management (*WAM*)

5.1.1 Water Cut

In the previous work in this research inter-well connectivity was measured based on twenty years of injection and production history. Section 3.4 explained how the producer’s water cut could be combined with the results of the inter-well connectivity measurements derived from the *CRM* technique to determine the water allocation factor for the next 20 years. The water cut is the parameter which changes during the well’s production life, after water breakthrough at the production wells. An alternation in the injection scenario by applying a new allocation factor to the injectors will change the water cut in the producers; hence the analysis in Section 3.4 has been extended by updating the injection allocation factor every 5 years, taking into account the history of change in the water cut after each 5-year interval.

Figure 3.1 shows the results of the new analysis, representing the water cut for each producer at the end of each 5 year interval.

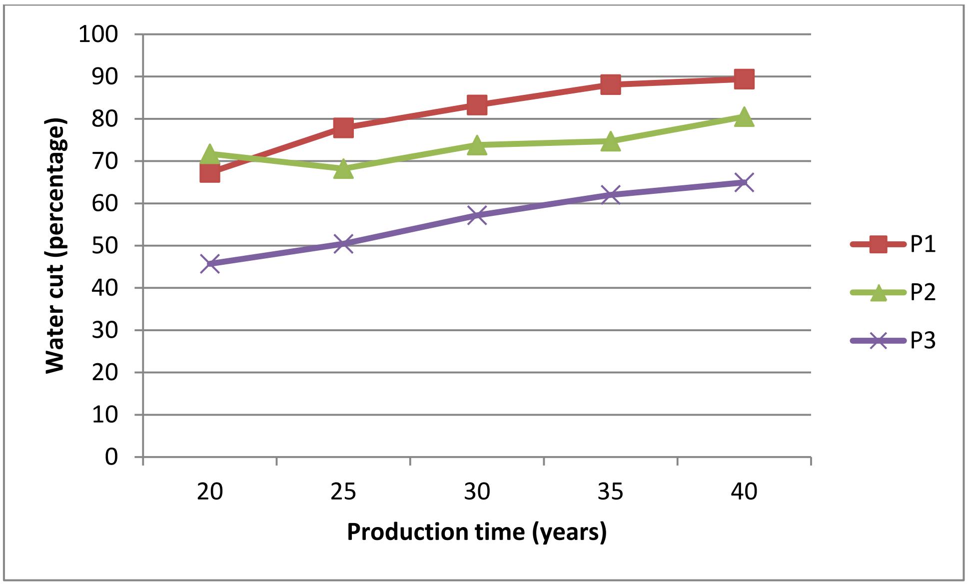

Figure 5. 1: Water cut of each producer at the end of each 5-year interval after beginning of the water allocation management.

Water cut is increasing in producers 1 and 3 while the water cut of producer 2 shows a decrease in the first 5 year period, then starts to increase with a similar trend as that of the other producers in the second period but again, in the third period, the water cut showed a smaller rate of increase compared with other producers. The water allocation factor for each of the four injectors, calculated from the results of *CRM* inter-well connectivity measurement and water cut of each producer (according to the procedure defined in section 3.4) is shown in Figure 3.2.

Figure 3.2 shows that injector 1 is consistently allocated the greatest volume of water while injector 3 receives the minimum fraction of available water. This is because producer 3, the best producer, receives most support from injector 1 (Figure 5.3).

Producer number 2 had the highest water cut at the beginning of the 20 year period of water allocation management. After 5 years its performance improved due to the efficiency of the water allocation management. This resulted in an increase in the volume of water allocated to the injectors connected to this producer during the second period. This can be observed in Figure 5.4, where more water allocated to the injector 4.

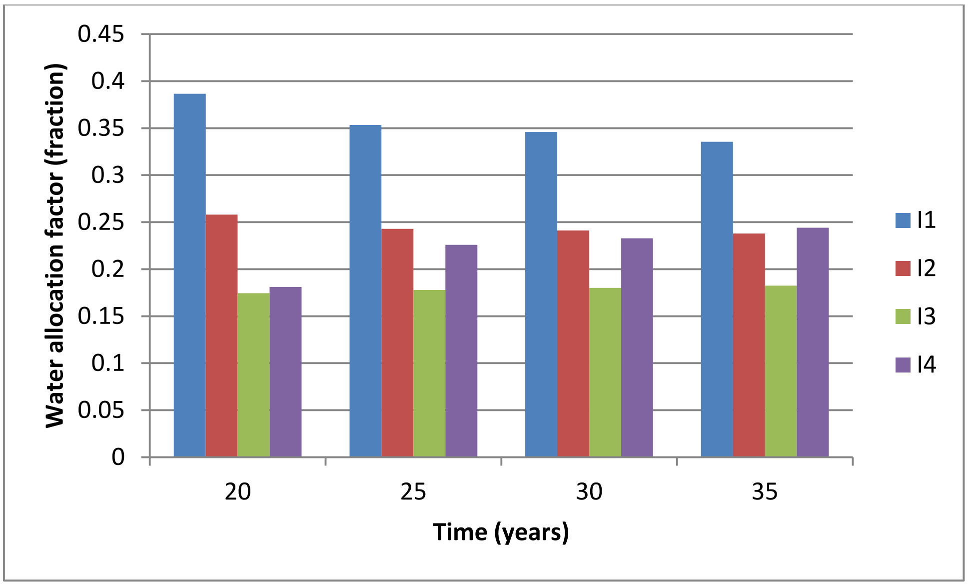

Figure 5. 2: Calculated water allocation factor for each injector at the end of each 5 year period.

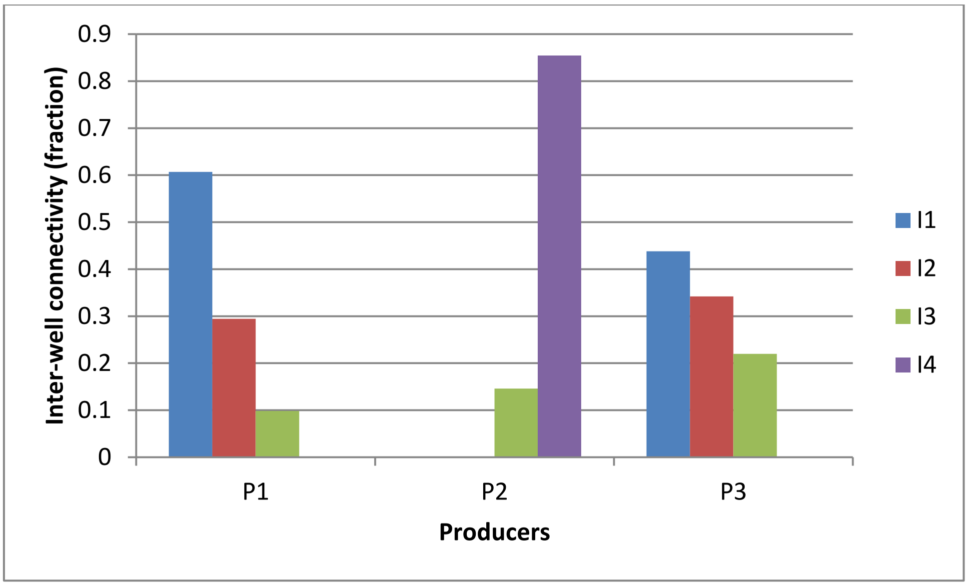

Figure 5. 3: Inter-well connectivity measurements determined from CRM for each producer.

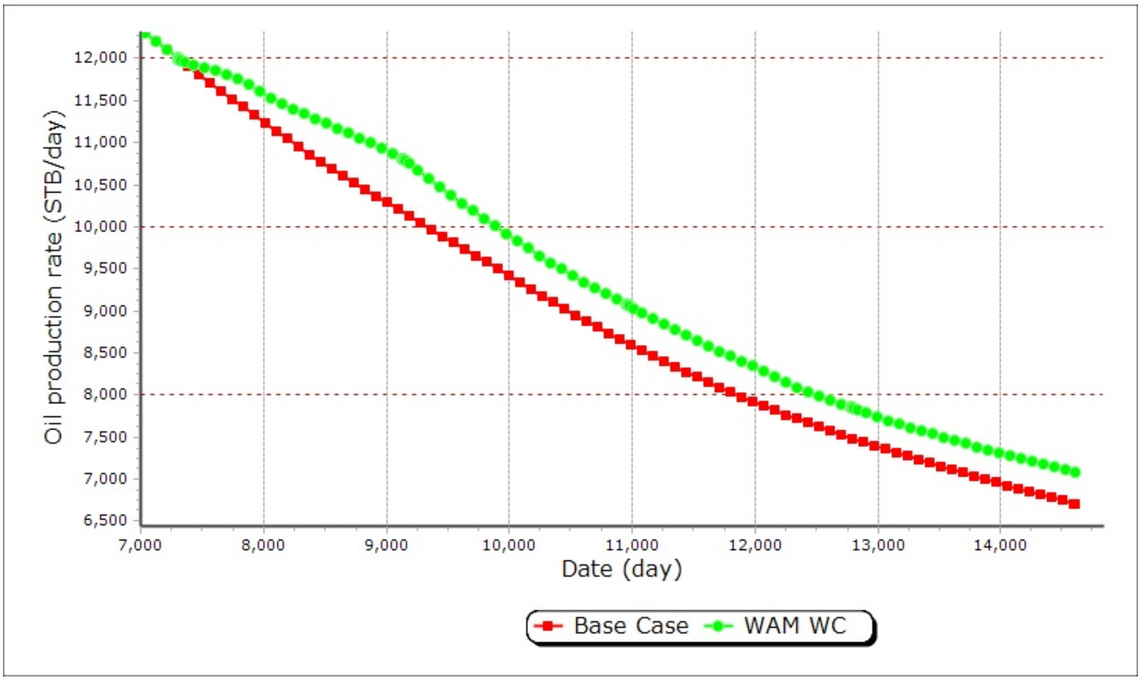

Figure 5. 4: Reservoir oil production rate for base case and the case with water allocation management.

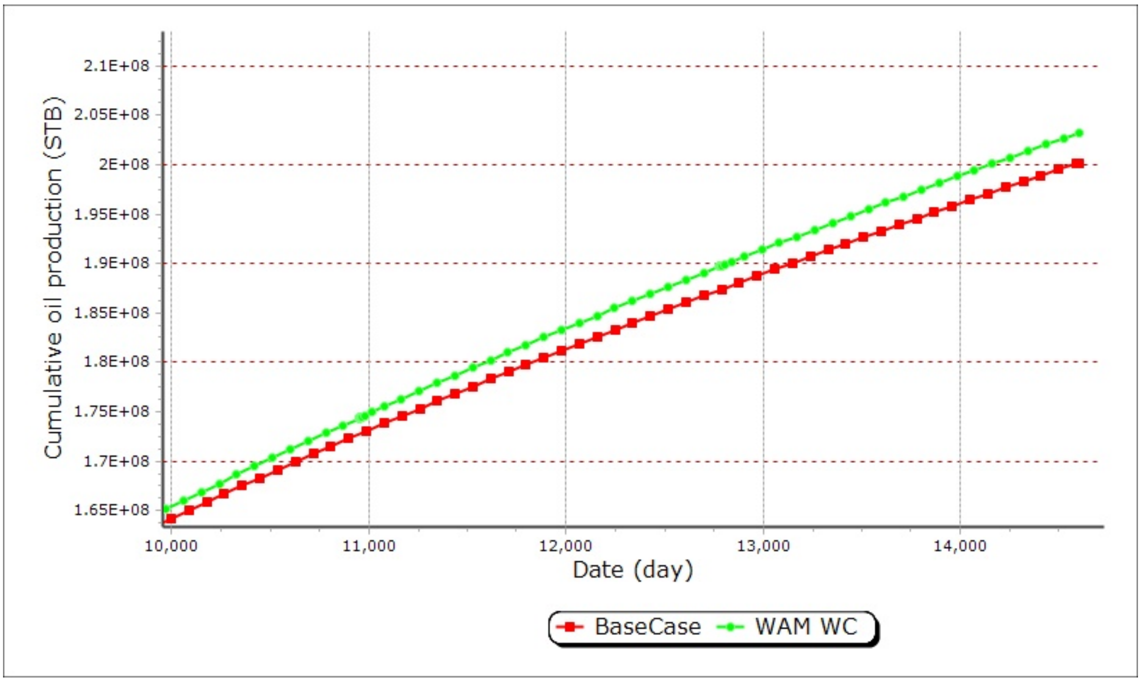

Figure 5. 5: Cumulative oil production for base case and the case with water allocation management.

Figure 5.4 compares the daily oil production profile between the base case, without allocation management, and the one with allocation management. It shows that the allocation increases oil production by 5%, resulting in an increase in the field cumulative oil production of 1.5% (Figure 5.5). Figures 5.6 and 5.7 illustrate how water production also decreased after water injection management was implemented.

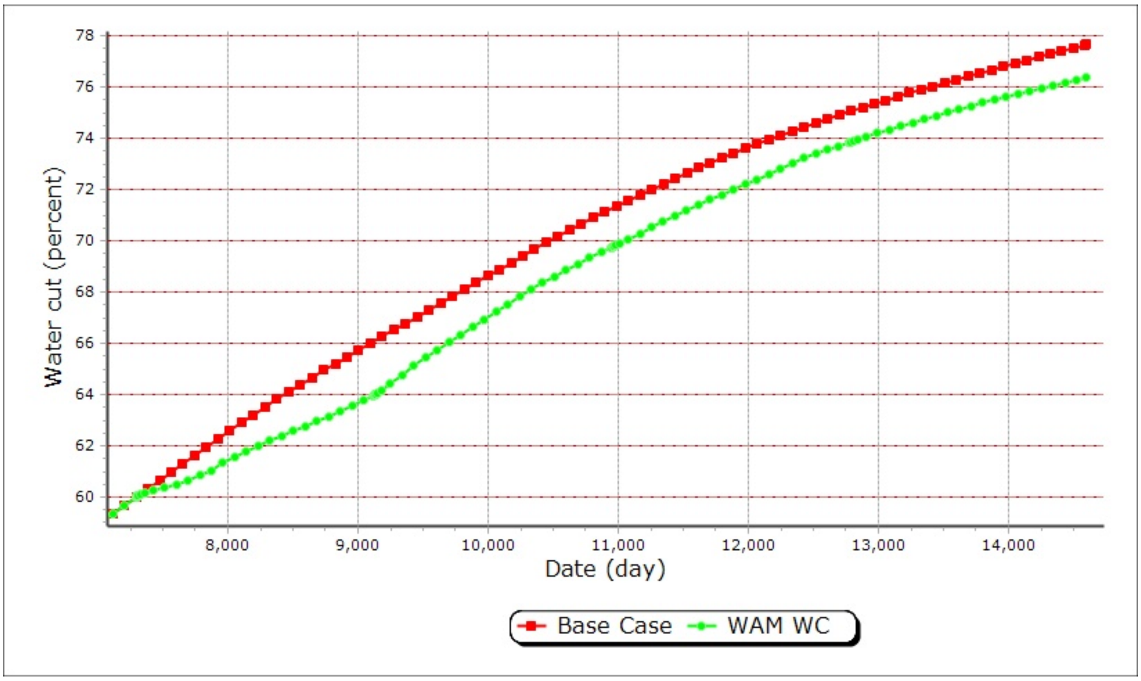

Figure 5. 6: Field water cut for base case and the case with water allocation management.

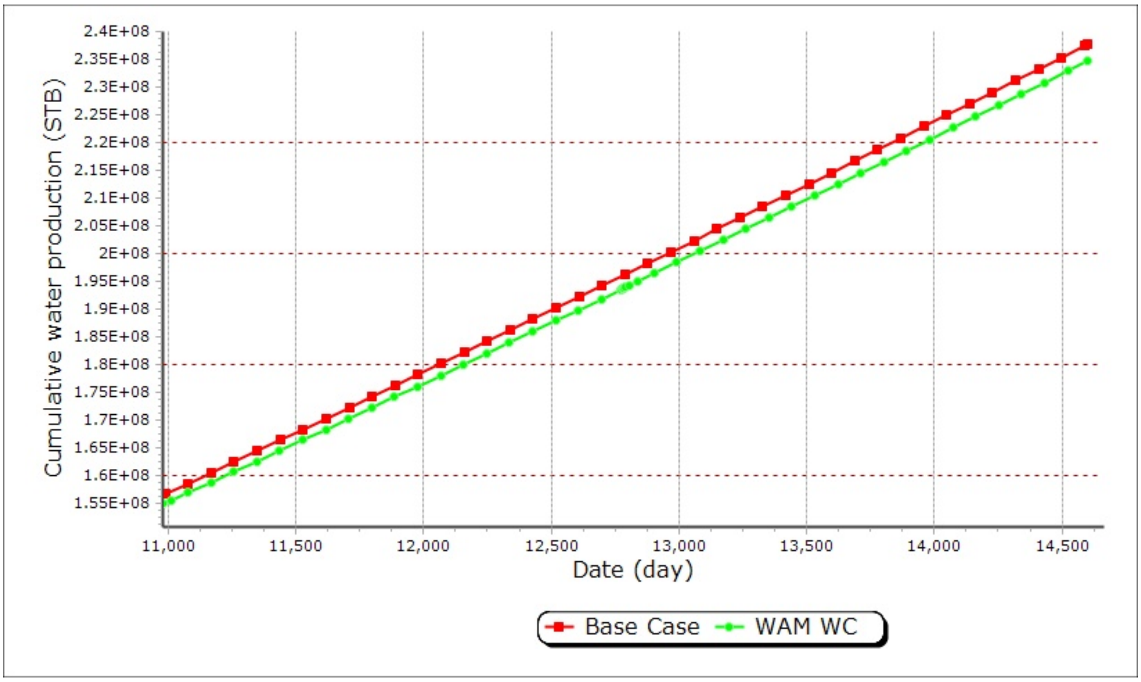

Figure 5. 7: Reservoir cumulative water production for base case and the case with water allocation management.

5.1.2 Cumulative Water Cut (CWC)

The previous analysis showed that the water cut can change significantly in a production well which is well-connected to the injectors, if the water allocation is changed without respecting the previous history of the production. This was illustrated by the experiment on producer number 2, in the previous section, when the water allocation factor based on the water cut was changed. It was concluded that this was not a suitable parameter for describing well performance when deciding long-term water allocation management. We therefore tested whether the cumulative water cut fraction (*CWC*) is a better parameter for water allocation management. *CWC* is defined as:

$$CWC = \frac{Q_w}{Q_l} \qquad \text{(Equation 5.1)}$$

in which Q_w and Q_l are the cumulative water production and the cumulative liquid production from a production well. Therefore Equation 3.21 in the water allocation management procedure described in Section 3.4 of Chapter 3will change to:

$$OI = (1 - CWC) \times LR \qquad \text{(Equation 5.2)}$$

The remainder of the procedure described in Chapter 3 remains unchanged. The following figure shows the change in *CWC* for each producer after carrying out water allocation management at 5-year intervals. Unlike the previous analysis, in this case, producer 2 is still the worst producer and therefore the calculated water allocation factor has not changed considerably (Figure 5.9).

Figures 5.10 to 5.13 report the improved results when applying *CWC* for allocation management rather than *WC*: a 10% increase in daily oil production, 2% increase in cumulative oil production and a 1.6% decrease in cumulative water production resulted. More oil was initially produced when water allocation was based on *WC* but *CWC* gave better results during the subsequent fifteen-year period. (Figures 5.10 to 5.13). It was concluded that *WC* is more suitable for short-term optimization of the oil production, while *CWC* is to be preferred for long term optimization.

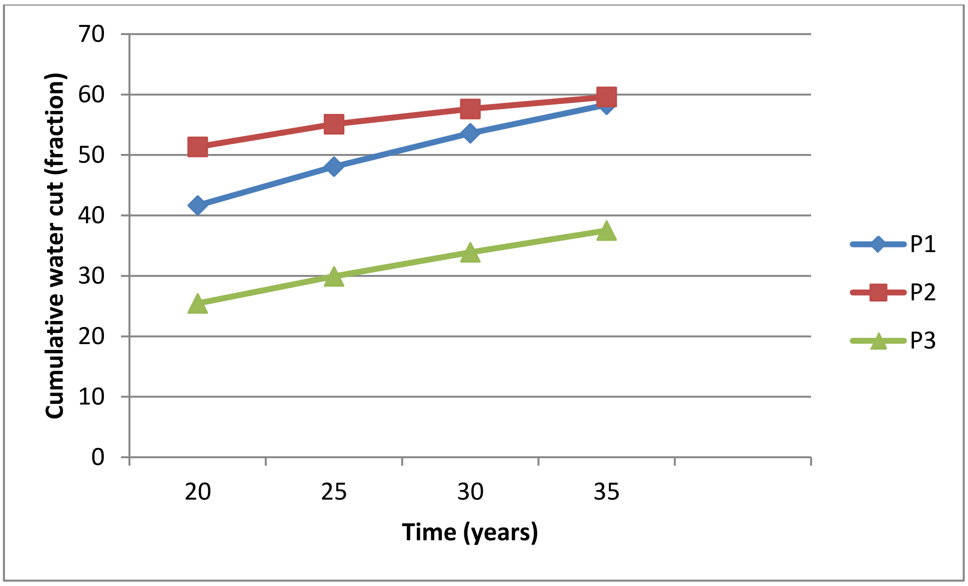

Figure 5. 8: Cumulative water cut of each producer.

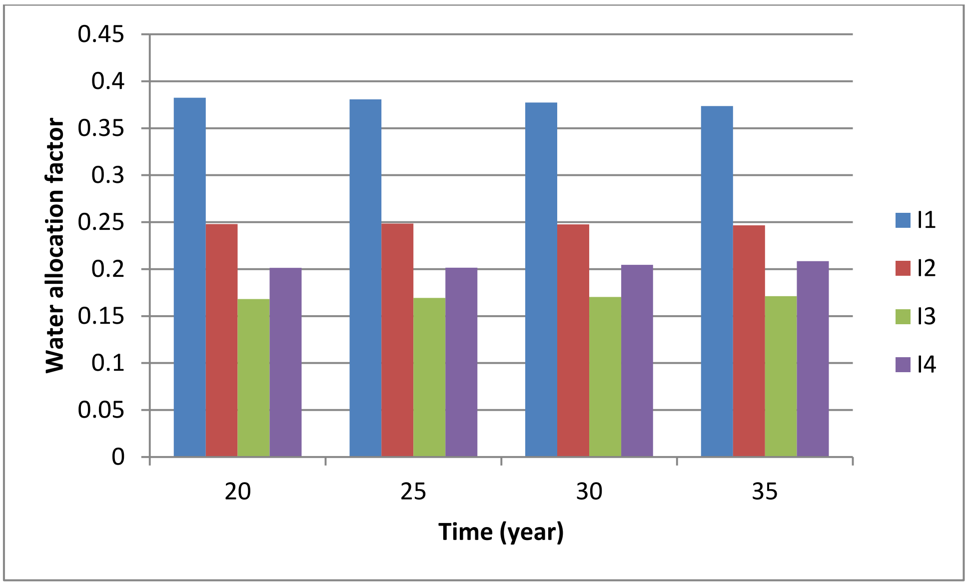

Figure 5. 9: Calculated water allocation factor for different production periods.

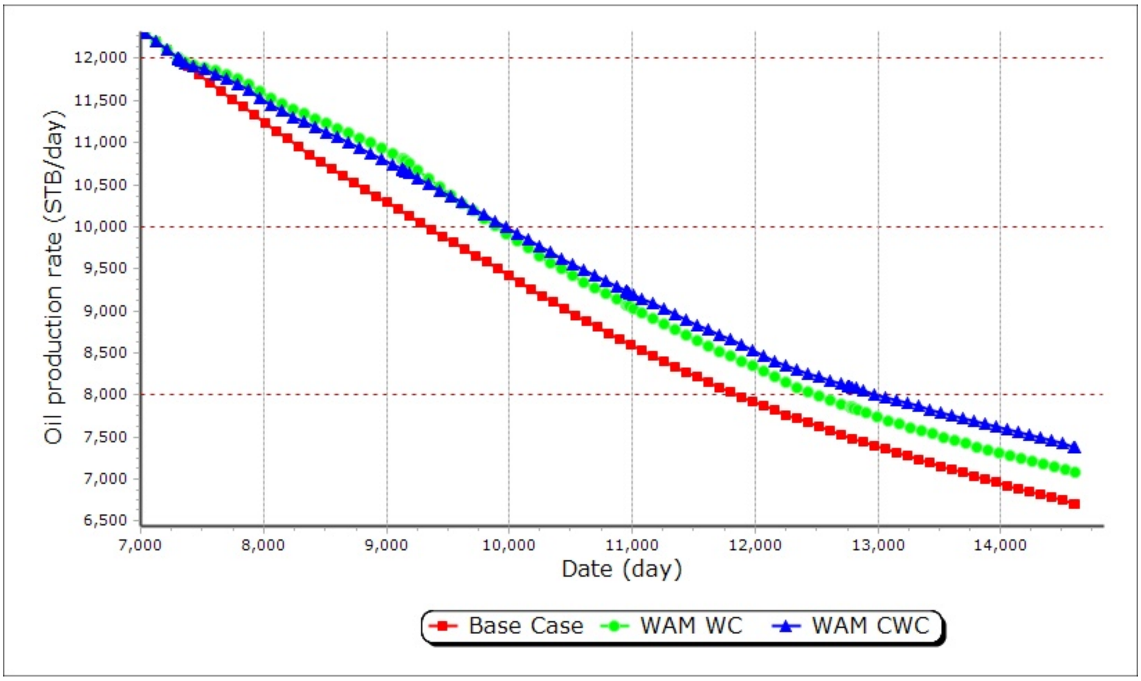

Figure 5.10: Reservoir oil production rate for all three injection scenarios.

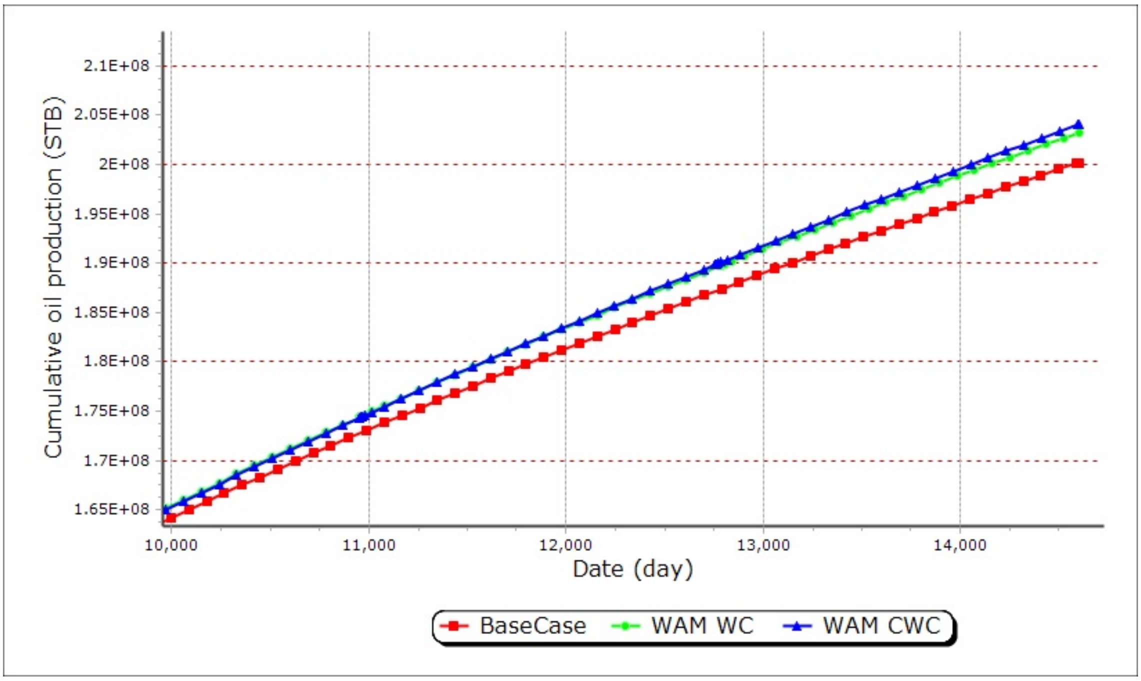

Figure 5.11: Reservoir cumulative oil production for all three injection scenarios.

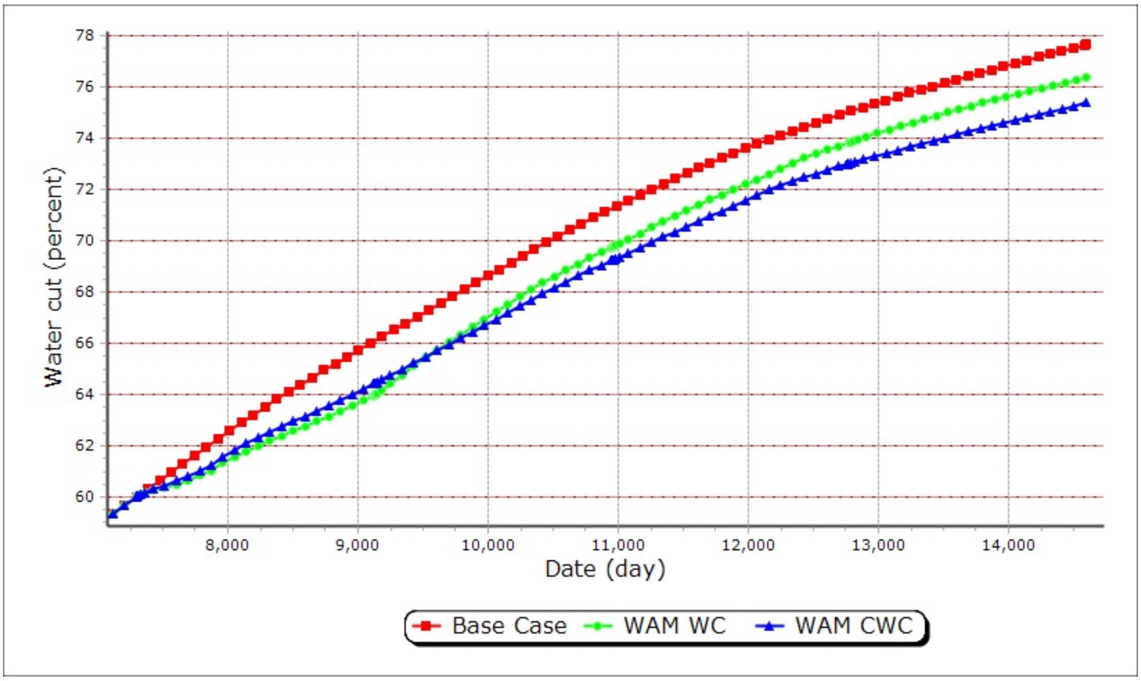

Figure 5. 12: Reservoir water cut for all three injection scenarios.

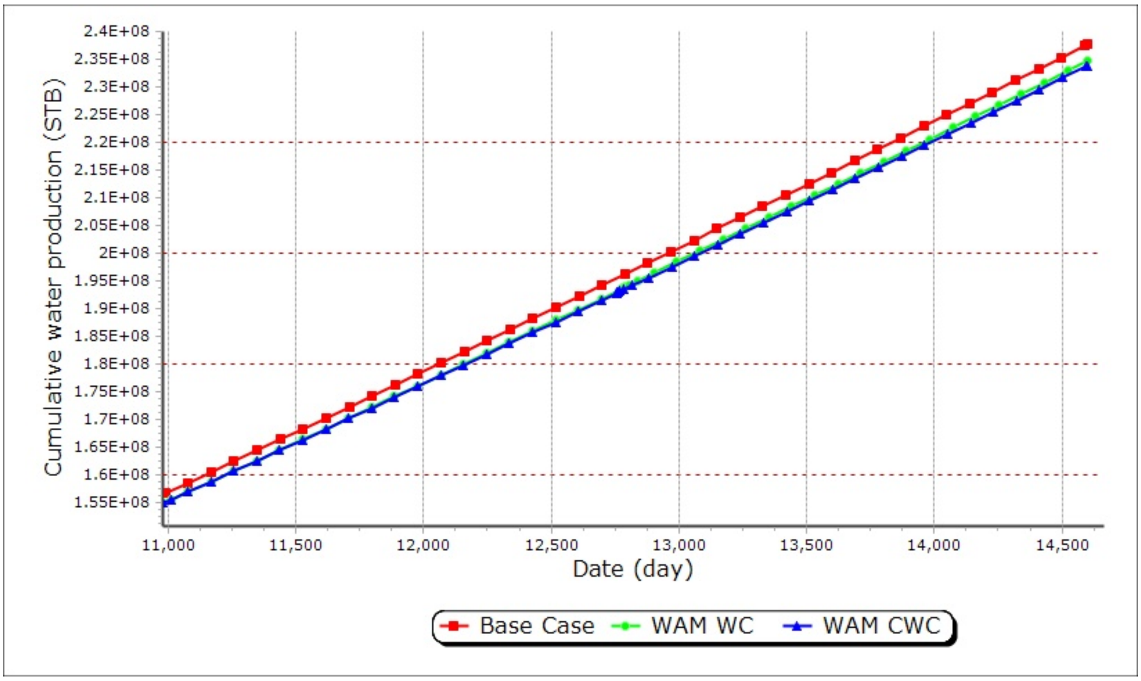

Figure 5.13: Field cumulative water production for all three injection scenarios.

5.1.3 Relative Oil Production Ratio (OPR)

The previous section showed that the CWC-based water allocation management gave better performance than a *WC*-based procedure, for long term optimisation. The *CWC* parameter has a value between 0 and 1, i.e., the worst *CWC* value, one that corresponds to a well with 100% water cut, is 0. Ideally, we would like to assign a negative value for a production well that produces more water than oil. A new parameter, the Cumulative Oil Production Ratio (*OPR*), to describe the production well's performance, has been defined as:

$$OPR = OR - WR \qquad \text{(Equation 5.3)}$$

In which *OR* and *WR*, the production well's Oil Ratio and Water Ratio respectively, are defined by:

$$OR = \frac{Q_o}{Q_{or}} \qquad \text{(Equation 5.4)}$$

$$WR = \frac{Q_w}{Q_{wr}} \qquad \text{(Equation 5.5)}$$

where Q_o and Q_w are previously defined as the well's cumulative oil and cumulative water production and the new parameters, Q_{or} and Q_{wr} are the reservoir's cumulative oil production and cumulative water production, respectively. Equation 5.3 shows that the *OPR* can have a negative value when a production well is producing more water than oil.

The oil index (*OI*) parameter used for water allocation management (Equation 3.21) now becomes:

$$OI = OPR \qquad \text{(Equation 5.6)}$$

Note that the *LR* parameter used in equations 3.21 and 5.2 may now be omitted, since the *OI* clearly includes a comparison of the individual well's production with that of the reservoir as whole. A consequence of the *OI* having a negative value is that the total water allocation index (AF_t), defined by equation 3.22, also has a negative value. The final allocation factor *(AF)* can be calculated from the following three combinations of AF_t values:

5.1.3.1 All AF_t have positive values

The final allocation factor, as defined by equation 3.24, may be used.

5.1.3.2 All AF_t have negative values

The final allocation factor is now:

$$AF_i = \frac{1-AF_{ni}}{\sum_{i=1}^{I}(1-AF_{ni})} \qquad \text{(Equation 5.7)}$$

In which AF_{ni} is the normalized allocation factor for the injector *i*:

$$AF_i = \frac{AF_{ti}}{\sum_{i=i}^{I} AF_{ti}} \qquad \text{(Equation 5.8)}$$

5.1.3.3 Both positive (AF_t^+) and negative (AF_t^-) values of AF_t are present

AF_n is now calculated separately for wells with positive and with negative AF_t values, and the final allocation factor will be:

1. For wells with positive AF_t:

   $$AF = \frac{1}{I}(1 + AF_{ni}^{+}) \qquad \text{(Equation 5.9)}$$

2. For the wells with negative AF_t:

   $$AF = \frac{1}{I}(1 - AF_{ni}^{-}) \qquad \text{(Equation 5.10)}$$

in which *I* is the number of injectors. The calculated allocation factor using the *OPR* approach for the four injectors at each of the four different periods is given in Figure 5.14. Figure 5.14 shows a significantly different water allocation factor to that calculated by the earlier approaches. Further, it does respond to the change in the production performance, especially towards the later time periods, when injector 2 shows a consistent increase in its water allocation factor.

Figure 5.15 is the *OPR* value for each producer. Two wells have a negative *OPR* value, resulting in a reduced volume of water being assigned to the injectors connected to those wells (Figure 5.14).

Applying this technique for water allocation management significantly improves the performance of water flooding compared with *WC* and *CWC* (see Figures 5.16 to 5.20). The main advantage of the *OPR* method is that it differentiates more clearly between good and bad producers|, being better able to "punish" injectors supporting producers with a high water cut.

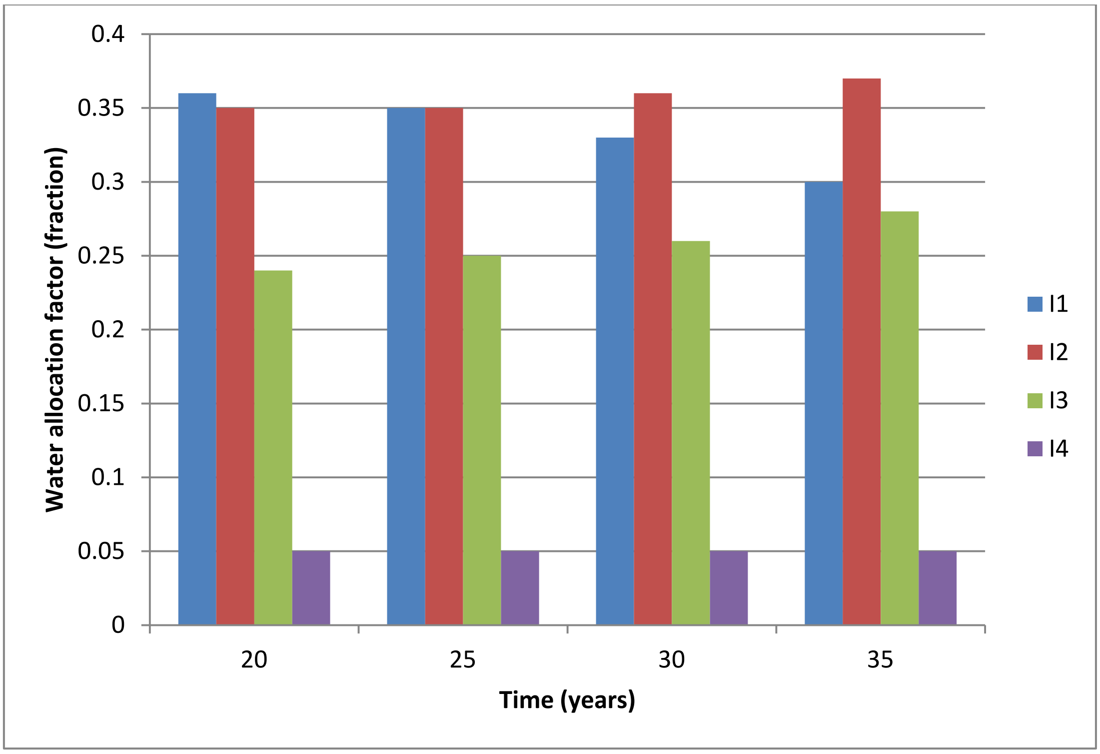

Figure 5. 14: Calculated water allocation factor based on OPR at the end of each 5-year period.

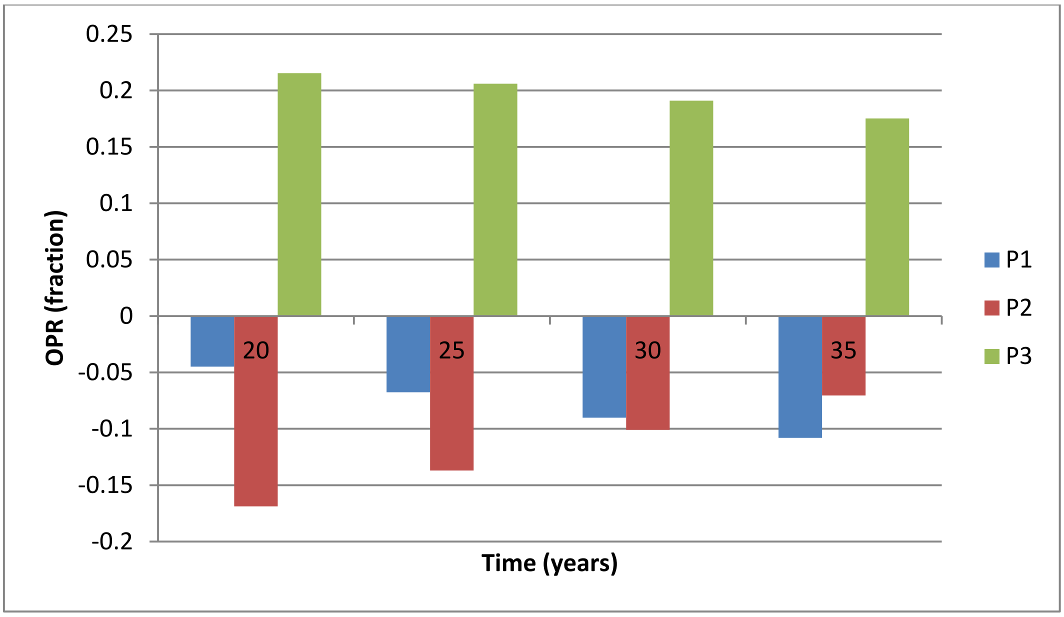

Figure 5. 15: Measured OPR for each producer at the end of each 5-year period.

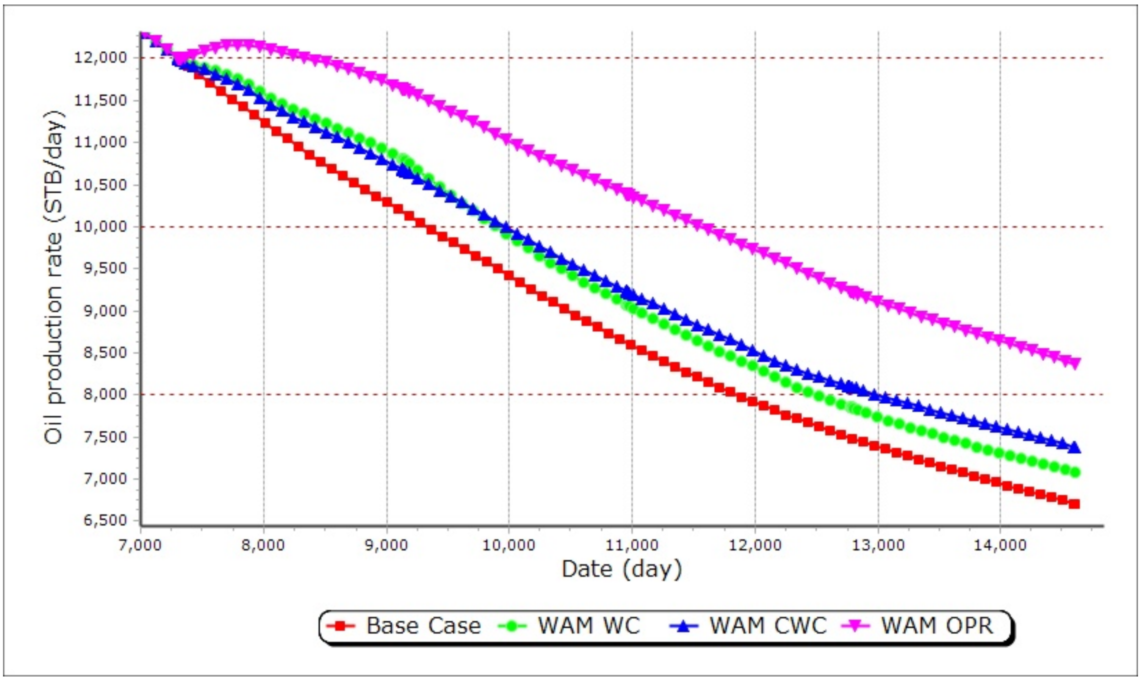

Figure 5. 16: Comparison of daily oil production rate obtained from different allocation management scenarios.

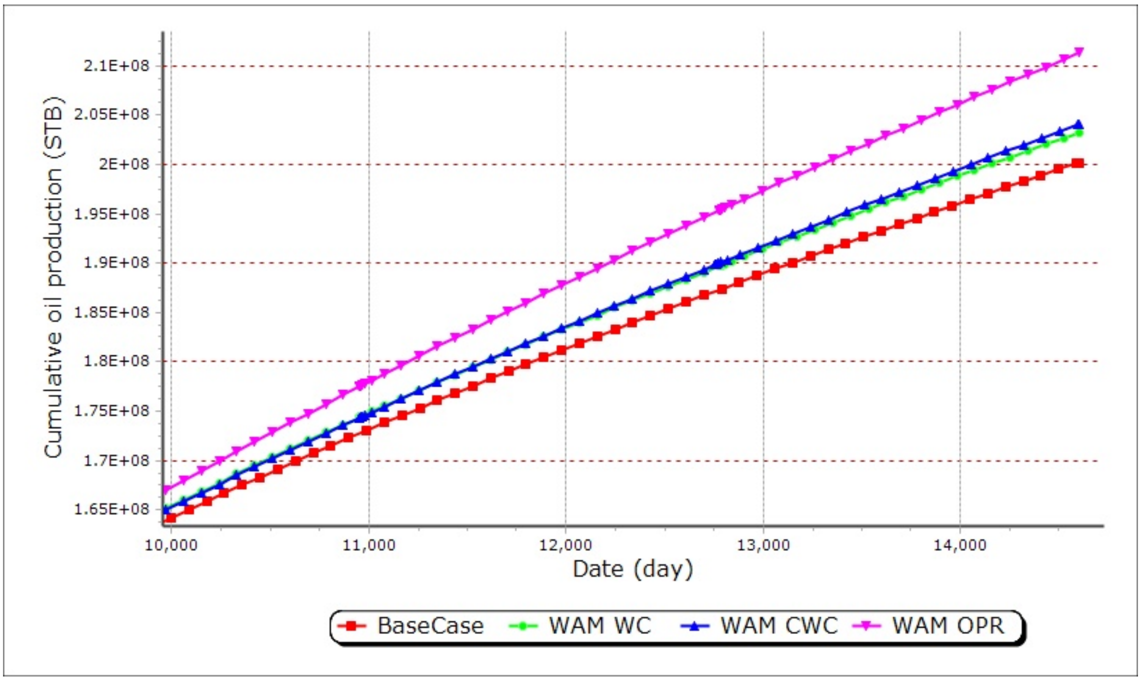

Figure 5. 17: Comparison of cumulative oil production obtained from different allocation management scenarios.

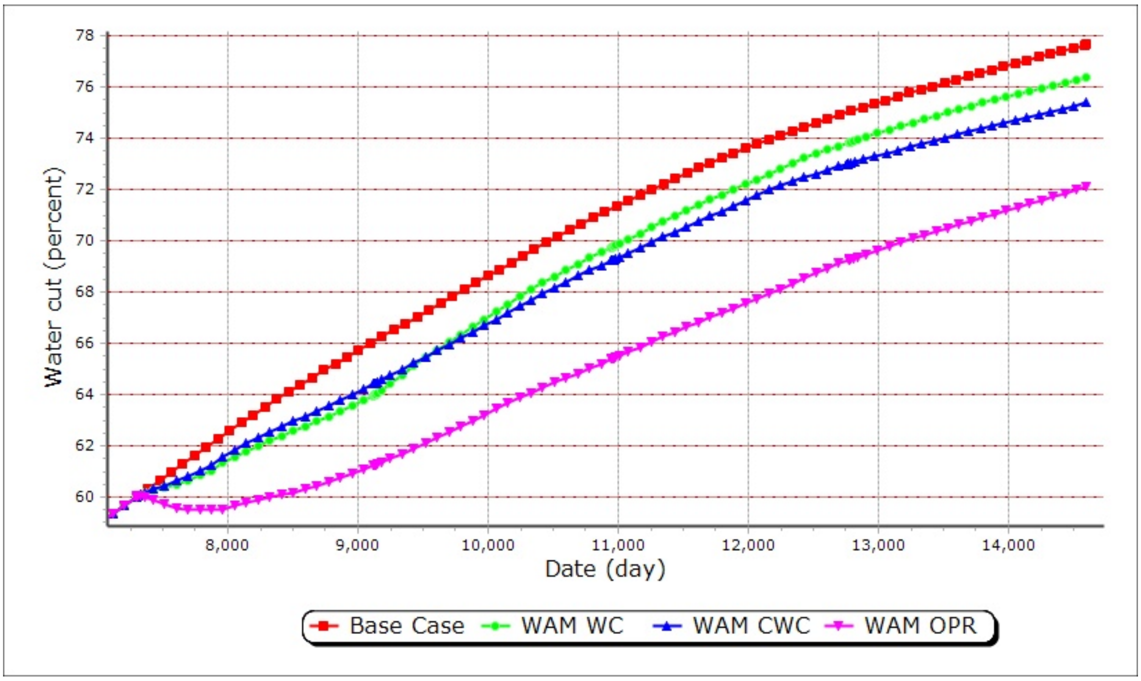

Figure 5. 18: Comparison of field water cut obtained from different allocation management scenarios.

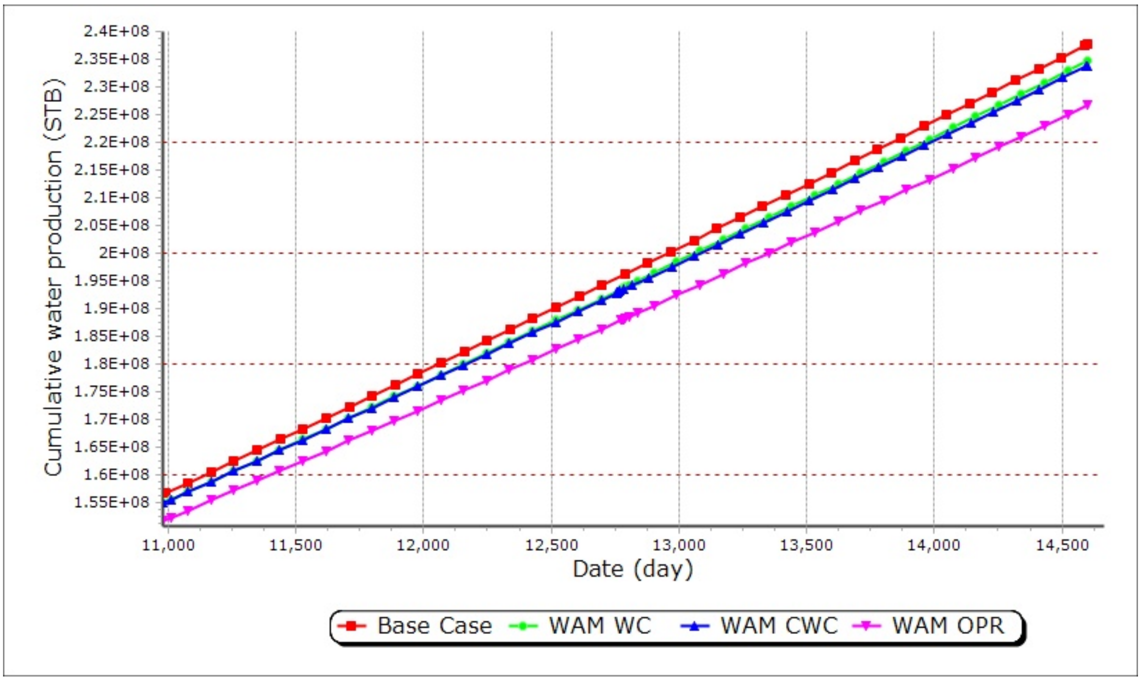

Figure 5. 19: Comparison of cumulative water production obtained from different allocation management scenarios.

5.1.4 Oil Production Index (OPI)

The range of *OPR* is between *-1* and 1 but it cannot be exactly *-1* or *+1*. For example, look at the example illustrated in Figure 5.18. This figure represents a reservoir consisting of 3 producers. For a certain period of time, one producer (*P1*) produced only oil, the second producer produced just water and the last producer produced with 75% *CWC*. Therefore, calculated *OPR* for these production wells will be *-0.571*, *0.8* and *-0.228* respectively.

In this study the aim is to assign -1 to a producer that is producing only water and 1 to the producer that producing only oil. This will help to improve the water allocation management. Therefore the new parameter is defined as the cumulative oil production index (*OPI*),

$$OPI = \frac{Q_o - Q_w}{Q_l} \qquad \text{(Equation 5.11)}$$

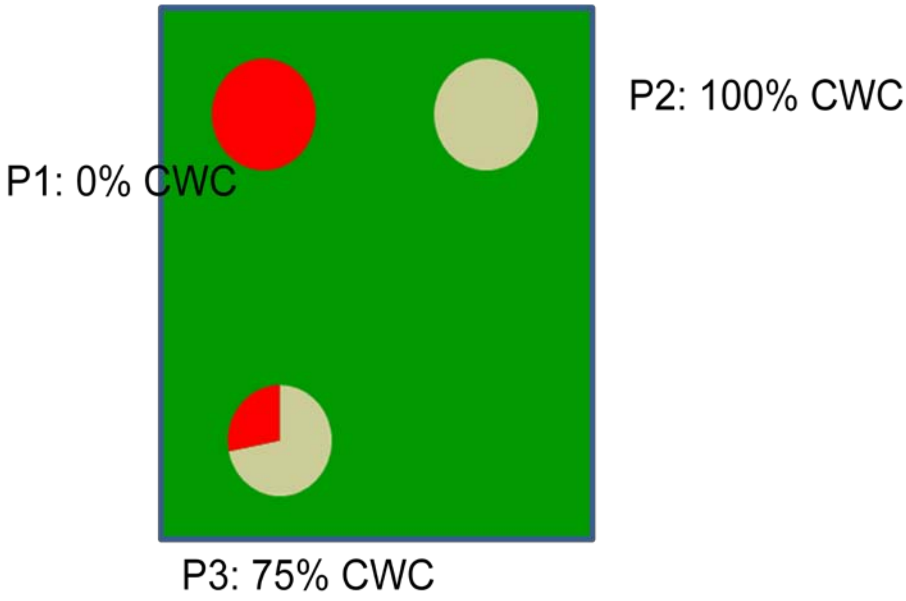

Figure 5. 20: Reservoir containing three production wells.

where Q_o and Q_w are previously defined as the well's cumulative oil and cumulative water production and the new parameter Q_l is cumulative liquid production. The *OPI* values for production wells in this example (Figure 20) will be +1 for *P1*, -1 for *P2* and -0.5 for *P3*.

Like *OPR*, *OPI* could be a negative value. Therefore, the allocation management procedure will be the same as for *OPR*. The only difference is the *OI* values for each producer. In this case. the Equation 3.21 will be like this:

$$OI = OPI_r - OPI \qquad \text{(Equation 5.12)}$$

where OPI_r and *OPI* are determined *OPI* values for reservoir and production well respectively. The calculated OPI for wells and reservoir in our case study is shown in Figure 5.21.

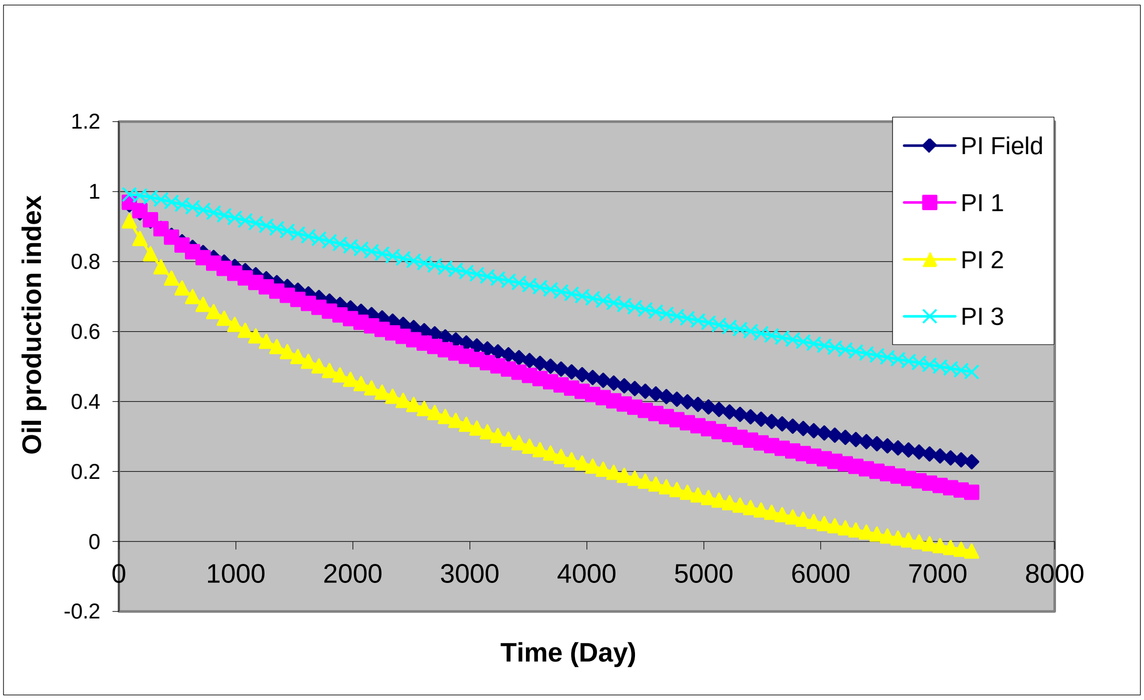

Figure 5. 21: Measured OPI for reservoir and each production well.

Figure 5.22 shows the determined water allocation factor for each period.

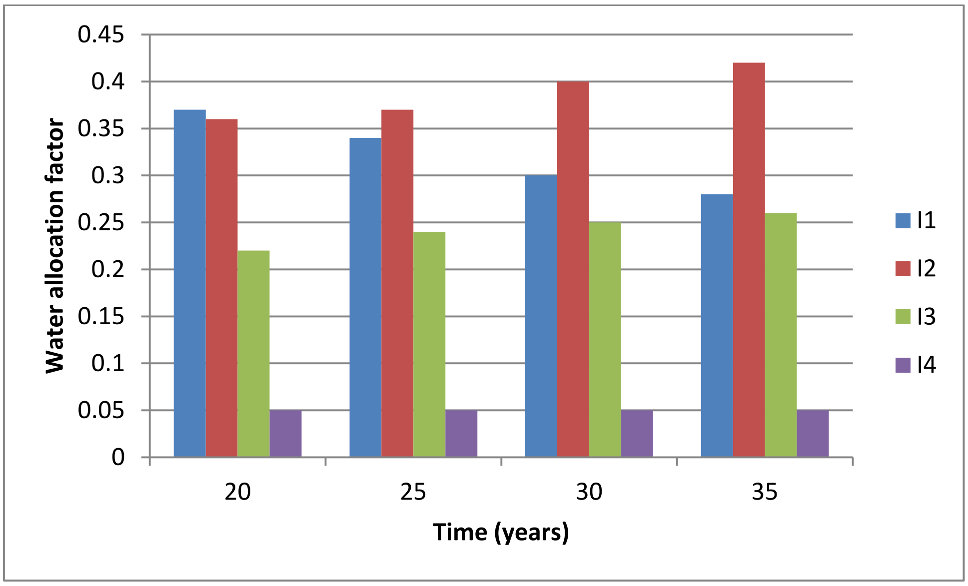

Figure 5. 22: Calculated water allocation factor, based on OPI at the end of each 5-year period.

Applying this technique for water allocation management slightly improves the performance of water flooding, when compared with *OPR* (see Figures 5.23 to 5.25).

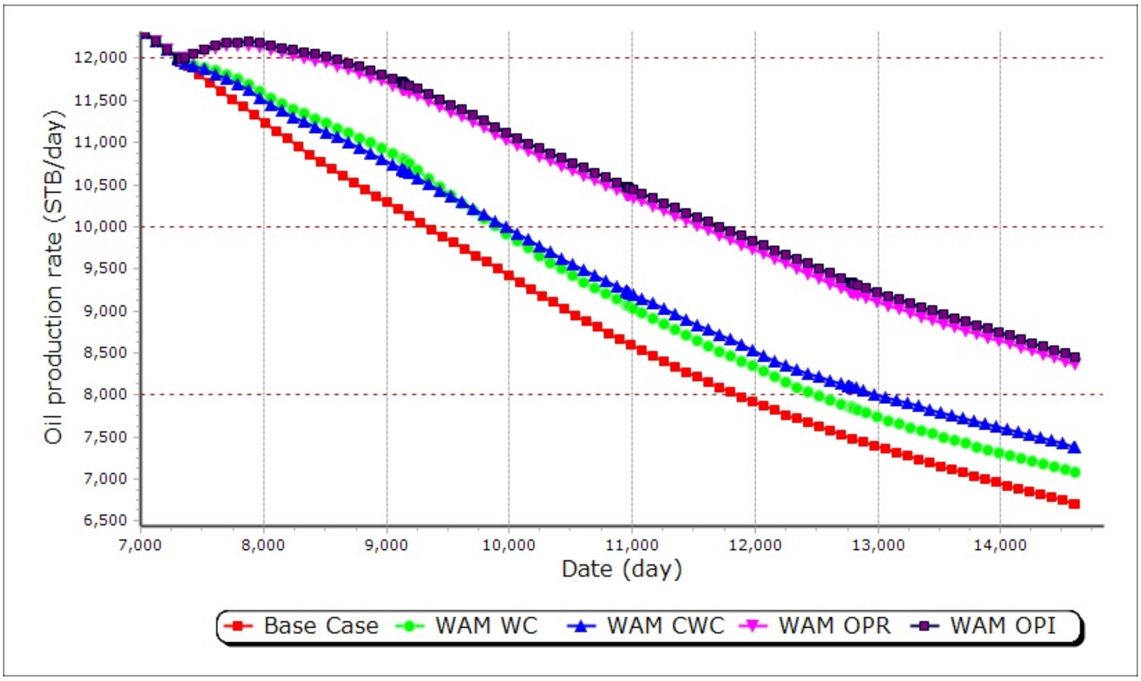

Figure 5. 23: Comparison of daily oil production rate obtained from different allocation management scenarios.

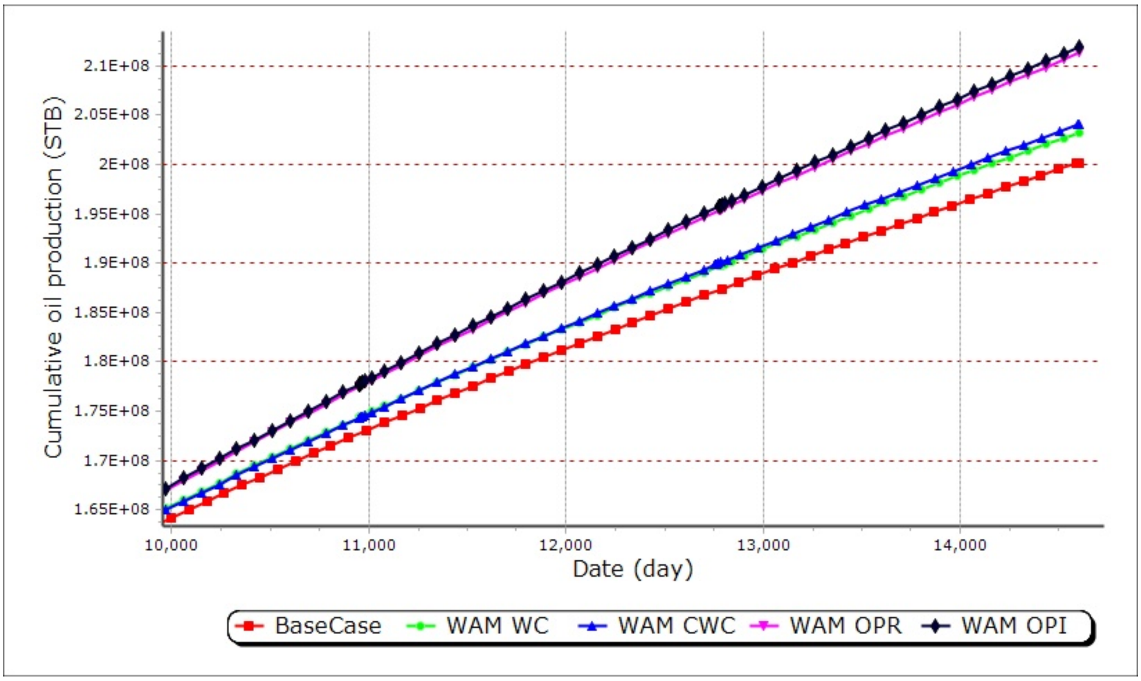

Figure 5. 24: Comparison of cumulative oil production obtained from different allocation management scenarios.

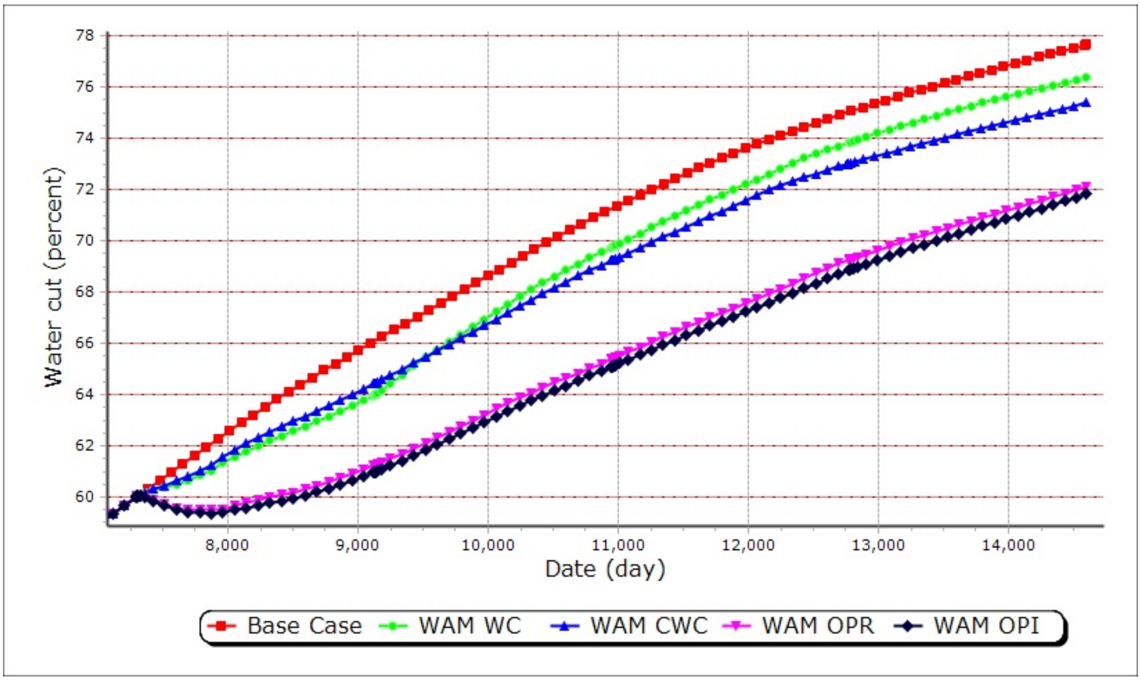

Figure 5. 25: Comparison of field water cut obtained from different allocation management scenarios.

Results of the allocation management with the water cut shows that water allocation factors may change considerably by changing the water allocation in the producers highly affected by injectors and also it did not respect the previous performance of the well. Better results were obtained from *CWC*. Although at the beginning *WC* worked better, *CWC* took over the *WC* at the final production intervals. Also the allocation factor obtained from *CWC* showed less tendency to change during different intervals by change in the performance of the production wells. Significantly better improvement was obtained from *OPR* and *OPI* compared to *CWC* (Figure 5.26).

Comparison of *WC* with *CWC* indicated that *WC* is good for short-term improvement while CWC can be applied for longer-term management. However, since *OPR* and *OPI* had more power to separate good and bad production wells, water allocation based on them helped to send more water to the injectors connected to the producers with higher oil production, while injecting less in the injector supporting production wells with higher water production. Because of that, significant improvement was achieved with allocation management with *OPR* and *OPI*, and *OPI* was found to work slightly better than *OPR*.

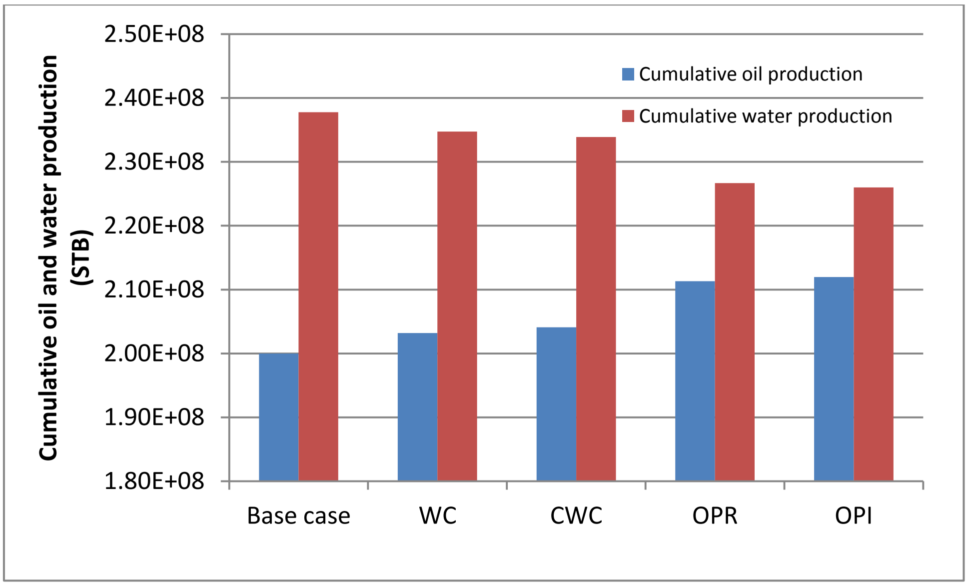

Figure 5. 26: Waterflooding performance in terms of cumulative oil and water production for different water allocation procedures.

5.2 Production Allocation Management (*PAM*)

In the previous section, we explained the different parameters we employed to describe the performance of the production wells for water allocation management. In this section we examine how the various parameters can be used to manage the allocation of production rate between the production wells, followed by evaluation of the improvement in the waterflood project performance in terms of more oil recovery and less water production. At first, we studied only production management with equal distribution of water between the injectors (no *WAM*). In the following sections, the parameters that have been used in this study will be reviewed.

5.2.1 Oil ratio (OR)

The *OR* is the first parameter to be employed for production allocation management. OR, defined by Equation 5.4, is the ratio of the cumulative oil production of a producer to the total cumulative oil production of the reservoir. Therefore, wells with higher *OR* have a larger share of the cumulative oil production from the reservoir. As a result, higher production rate should be allocated to the wells with higher *OR*. Equation 5.13 shows the calculated production allocation factor from *OR*.

$$PAF_j = \frac{OR_j}{\sum_{j=1}^{P}(OR_j)} \qquad \text{(Equation 5.13)}$$

In which PAF_j and OR_j are the production allocation factor and oil ratio for the producer, *j*, and *P* is the number of production wells.

5.2.2 Relative oil production ratio (OPR):

Normally a well will be producing oil but will also produce significant volumes of water as well. The relative oil production ratio (*OPR)* can be employed to define a production rate allocation factor that also takes into account the water production from the well. This technique will favour production from a well that produces more oil and less water.

Calculation of the *OPR* for each production well, using Equation 5.3, results in one of these three possibilities:

5.2.2.1 All OPR values are positive

In this case, the following equation can be used to determine the *PAF* for each producer:

$$PAF_j = \frac{OPR_j}{\sum_{j=1}^{P}(OPR_j)} \qquad \text{(Equation 5.14)}$$

in which OPR_j is the relative oil production ratio of the producer, *j*.

5.2.2.2 All OPR values are negatives

Production wells with negative values are producing more water than oil. In such a condition, Equation 5.15 shows how *PAF* can be calculated.

$$PAF_j^- = \frac{(1-OPR_j^-)}{\sum_{j=1}^{P^-}(OPR_j^-)}$$
(Equation 5.15)

where PAF_j^- and OPR_j^- are the production allocation factor and relative oil production ratio of the producer, *j* (the minus sign refers to the well having negative *OPR* value). P^- is the number of production wells with *OPR* less than 0.

5.2.2.3 Both positive (OPR_j^+) and negative (OPR_{jt}^-) values of OPR are present

PAF will be calculated separately for wells with OPR_j^- *and wells with* OPR_j^+.

$$PAF_j^- = \frac{OPR_j^-}{\sum_{j=1}^{P^-}(OPR_j^-)}$$
(Equation 5.16)

$$PAF_j^+ = \frac{OPR_j^+}{\sum_{j=1}^{P+}(OPR_j^-)}$$
(Equation 5.17)

where PAF_j^+ and OPR_j^+ are the production allocation factor and relative oil production ratio of the producer, *j*. P^+ is the number of production wells with *OPR* more than 0. In the next step the production rates of the negative wells will be decreased and at the same time the production rate of producers with positive *OPR* will be increased. The following equations introduce the new production rates of each type of well.

$$q_{nj}^- = q_{oj}^- - (PAF_j^- \times q_{oj}^-)$$
(Equation 5.18)

$$q_{nj}^+ = q_{oj}^+ + (PAF_j^+ \times q_{oj}^+)$$
(Equation 5.19)

q_{oj}^- and q_{nj}^- are the old and new production rate of the production well, *j*, that has a negative value for *OPR* . q_{oj}^+ and q_{nj}^+ correspond to the old and new production rates of producers having positive *OPR*.

5.2.3 Oil production index (OPI)

OPI, which has been previously defined by Equation 5.11, is final parameter tested for production allocation management. In water allocation management we found that *OPI* is the best parameter for describing production well performance, as it differentiates clearly between a good producer and a bad producer. For determining the production allocation for each production well, the *OPI* of each individual is compared with the *OPI* of the reservoir.

$$OPI_{rj} = OPI_j - OPI_r \qquad \text{(Equation 5.20)}$$

where OPI_{rj} is the difference between producer OPI_j and reservoir OPI_r . Therefore, there will be two types of the wells: wells with $OPI_{rj} > 0$ and producers with $OPI_{rj} < 0$.

The *PAF* for well wells with OPI_{rj} less than 0 will be calculated as:

$$PAF_j^- = \frac{OPI_{rj}^-}{\sum_{j=1}^{P^-}(OPI_{rj}^-)} \qquad \text{(Equation 5.21)}$$

The production rate of the producers having a negative effect on the reservoir *OPI* will then be reduced, based on Equation 5.21.

$$q_{nj}^- = q_{oj}^- - (PAF_j^- \times q_{oj}^-) \qquad \text{(Equation 5.21)}$$

The production allocation factor (PAF_j^+) of positive OPI_{rj} (OPI_{rj}^+) is calculated using equation 5.22.

$$PAF_j^+ = \frac{OPI_{rj}^+}{\sum_{j=1}^{P^+}(OPI_{rj}^+)} \qquad \text{(Equation 5.22)}$$

The amount of production rate subtracted (q_s^-) from producers with a negative effect on the reservoir *OPI* (OPI_{rj}^-) will be calculated by Equation 5.23.

$$q_s^- = \sum_{j=1}^{P^-} q_{oj}^- - \sum_{j=1}^{P^-} q_{nj}^- \qquad \text{(Equation 5.23)}$$

q_s^- will then be added to the production rate of the producers with OPI_{rj}^+, according to the following equation.

$$q_{nj}^+ = q_{oj}^+ + (PAF_j^+ \times q_s^-) \qquad \text{(Equation 5.24)}$$

Figures 15.27 to 15.29 represent the results of the production allocation management from all three methodologies. The studied model is the same as the one used for water allocation management. The first 20 years of production history are used to define new a production allocation for each producer for the next 20 years. The process of production allocation management is updated every 5 years. Minimum and maximum liquid production is introduced for each production well, based on the sensitivity analysis on the well models.

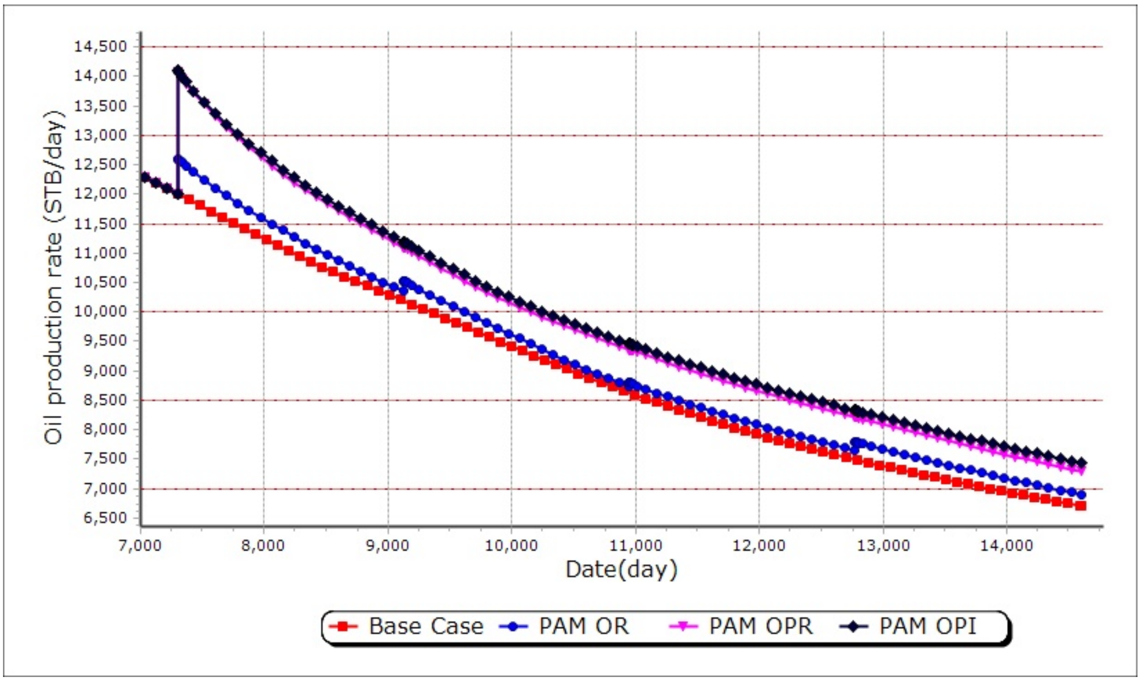

Figure 5. 27: Oil production rate obtained from different techniques of production allocation management.

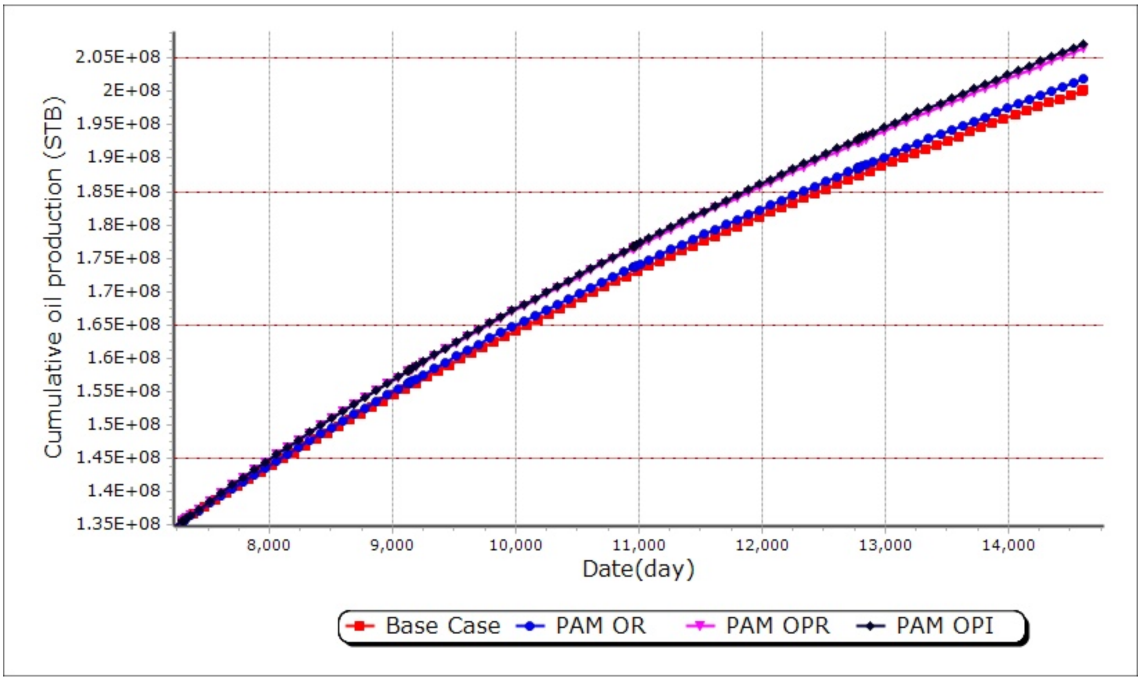

Figure 5. 28: Cumulative oil production obtained from different techniques of production allocation management.

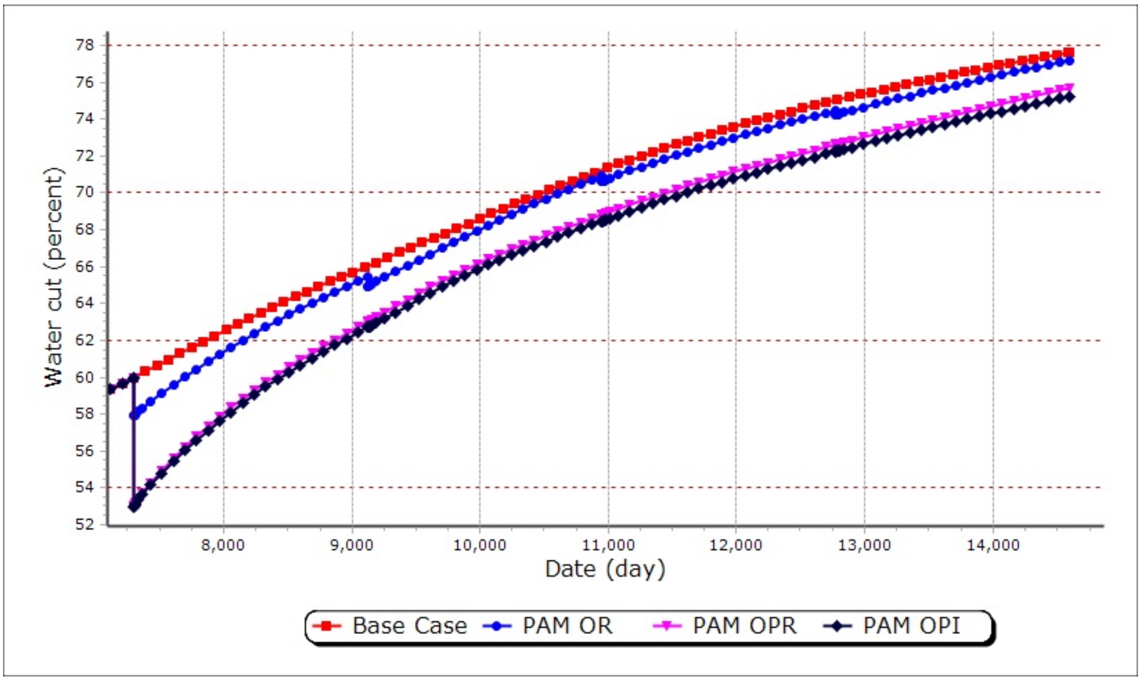

Figure 5. 29: Produced water rate obtained from different techniques of production allocation management.

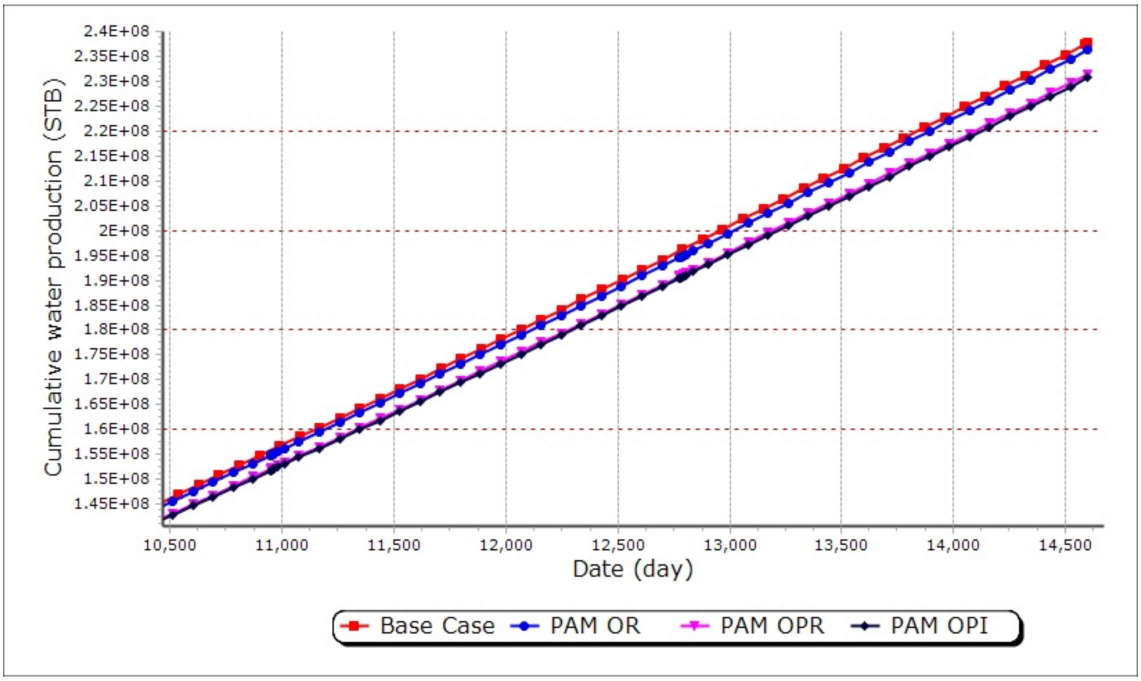

Figure 5. 30: Cumulative water produced from different techniques of production allocation management.

As expected, *PAM* caused more oil and less water to be produced. Significant improvement was obtained from *OPR* and *OPI*. As discussed in *WAM*, these two methods, although they not only tend to increase oil production from producers that are producing oil but they also apply a penalty if the well also producing water. Since *OPI* introduces a greater difference between a good producer and bad producer, more improvement is achieved from it.

As both *WAM* and *PAM* increased the efficiency of the studied waterflooding project, it was therefore decided to apply both types of allocation management simultaneously.

5.3 Production and injection allocation management (*W&P AM*)

CRM results of inter-well connectivity measurement and *OPI* are employed, based on the procedures explained in sections 5.1.4 and 5.2.3 to define the water and production allocation factor for the injectors and producers at the same time. The results of this analysis are shown in the following figures.

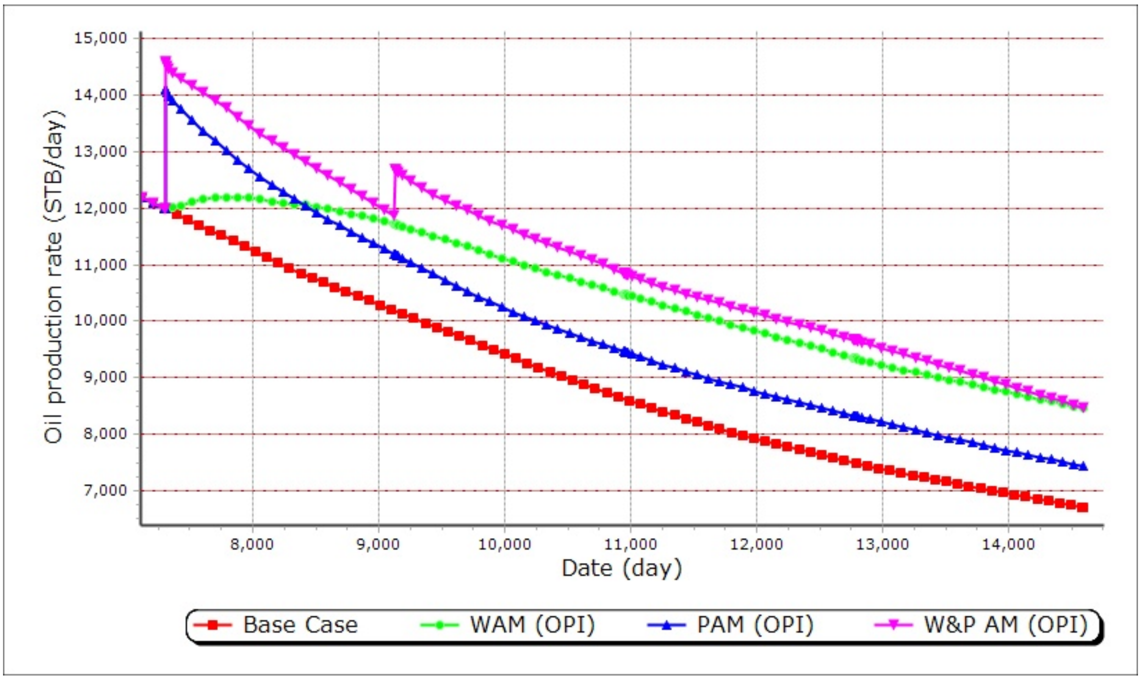

Figure 5.31: Comparison of oil production rate obtained from WAM, PAM and W&P AM.

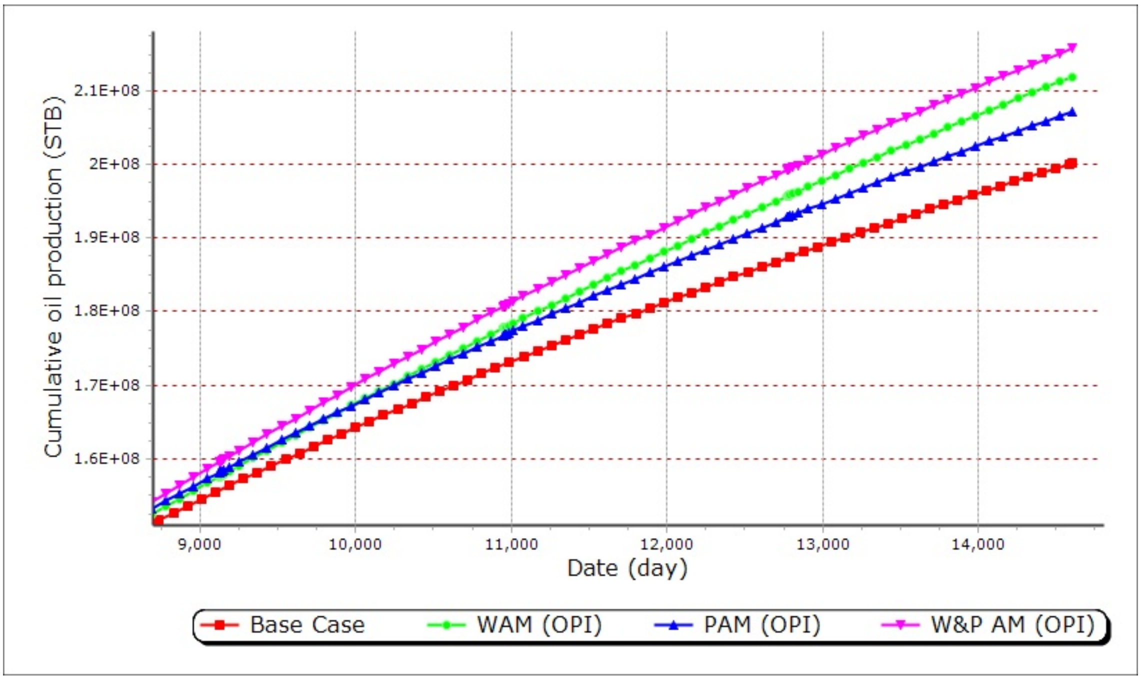

Figure 5. 32: Cumulative oil production from WAM, PAM and W&P AM.

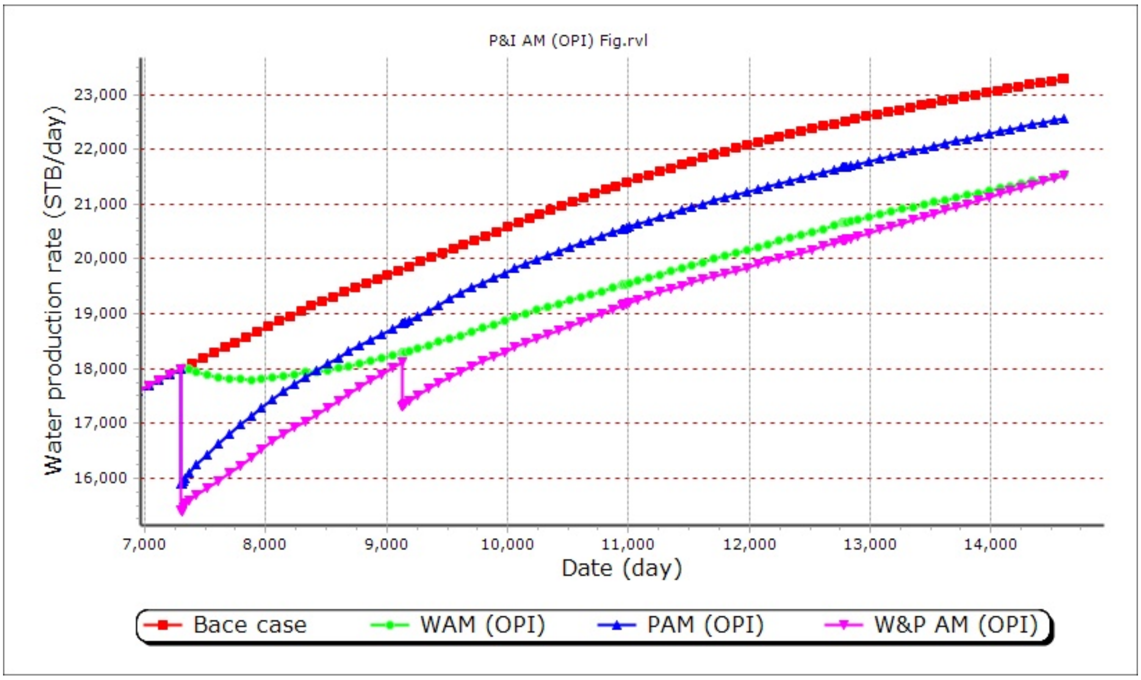

Figure 5. 33: Water production rate from WAM, PAM and W&P AM.

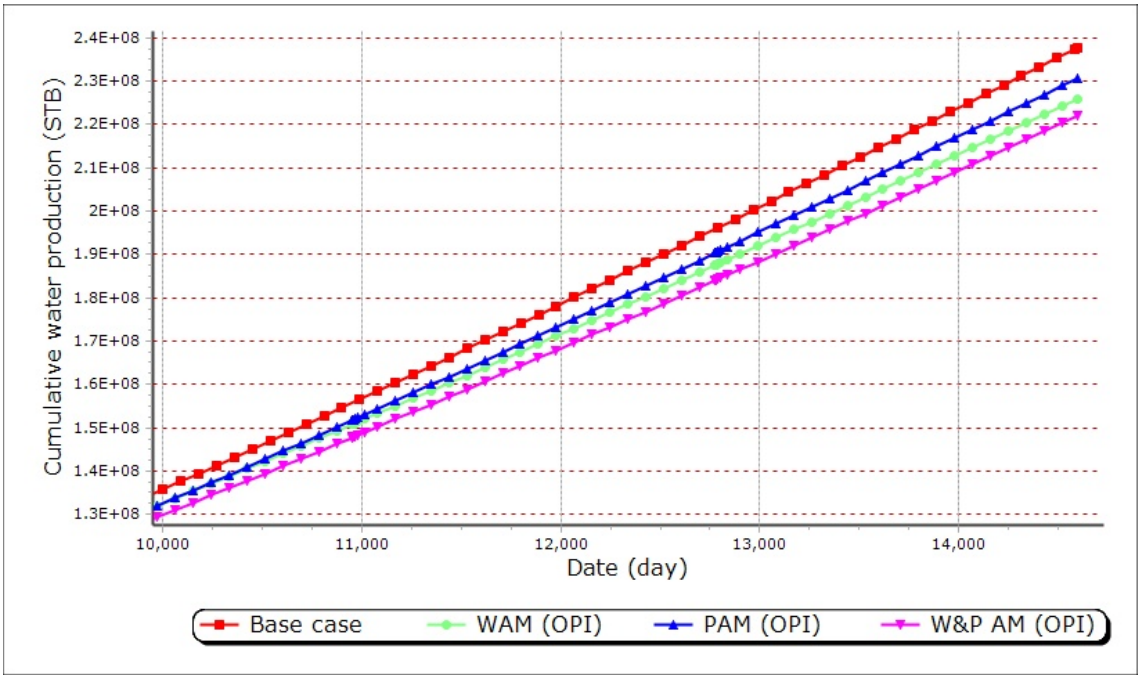

Figure 5. 34: Cumulative water production obtained from WAM, PAM and W&P AM.

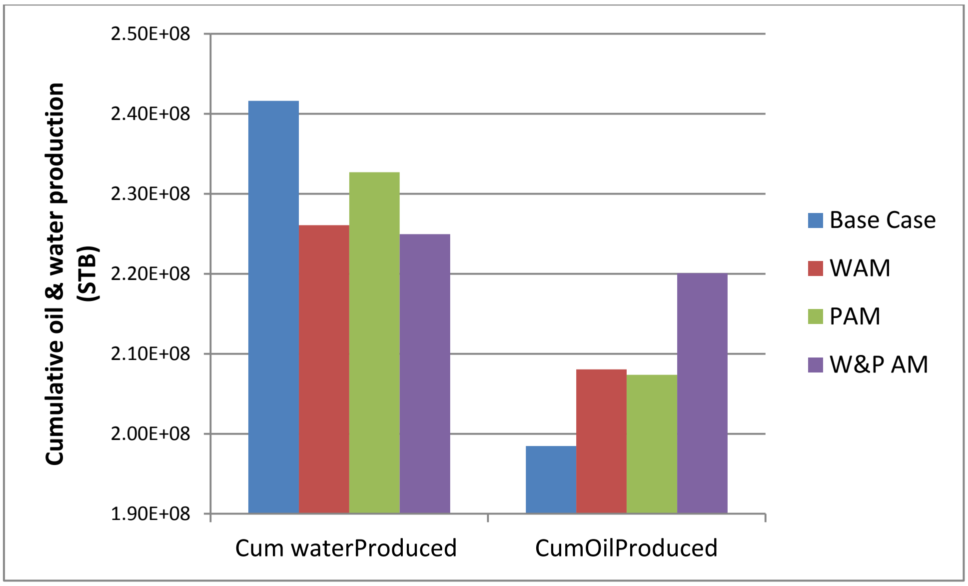

Figure 5. 35: Bar chart of cumulative oil and water production from WAM, PAM and W&P AM.

The comparison of *PAM* and *WAM* shows that *WAM* is more effective in improving waterflood efficiency than *PAM*. Better oil production was obtained at the beginning of *PAM*, while the improvement built up slowly in the case of *WAM*. This is because there will be a lag time in seeing the change in the production rate as a result of change in the injection scheme. The reduction in water production was bigger than the increase in oil production when changing from *WAM* to *PAM*, showing that a combination of inter-well connectivity and production performance monitoring were good for determining the water allocation factor.

W&P AM significantly improved the performance of the project. However, it was much more effective in increasing cumulative oil production (the amount of improvement was twice that of *WAM* and *PAM*) than in decreasing water production (Figure 5.33). This is because the both *WAM* and *PAM* are good at improving the oil production rate, while *PAM* is not very good at decreasing water production. This is because *PAM* increases the liquid production rate of the good producers. So due to increase in the liquid production of these wells, water production also will increase.

Although new techniques have been developed for production allocation management but in a reservoir, liquid and oil production are often constrained by the reservoir conditions, outflow performance of the wells, flow characteristics of the pipeline network, fluid-handling capacity of surface facilities, safety and economic considerations, or a combination of these factors. While adjusting well production rates and allocating water-injected rates can control production, implementation of these controls in an optimal manner is also very important and is not easy. Determination of the optimal operational settings at a given time, subject to all constraints, to achieve certain operational goals is the objective of dynamic production optimization, which requires simultaneous consideration of the interactions between the reservoir, the wells, and the surface facilities [10].

5.4 References

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Chapter 6– Water Allocation Optimisation

6.1 Introduction

In the previous chapters it was shown that water allocation was managed by combining the results of connectivity measurements with performance evaluation of production wells. In that methodology, less water was allocated to the injectors that were well-connected to the high water cut producers.

This chapter will propose another approach for water allocation management. This involves monitoring the effect of a change in the injection rate on the produced oil in the producers. We will then employ an optimisation engine to find the best injection rates for maximising the oil production rates.

We can forecast production rates by determining the injection rates of injection wells, for example by multi-linear regression (*MLR*), capacitive resistive model (*CRM*) and the reservoir simulator. In the next section *MLR* and *CRM* will be reviewed again in order to find a suitable optimiser tool that can be connected to them for defining best allocation factors for injection wells.

6.2 *MLR* review

Let the liquid production rate q_j from the well j be described by the following linear model in terms of the injection rates:

$$q_j = \sum_{j=1}^{I} \beta_{ij} i_i, \qquad \text{(Equation 6.1)}$$

where the coefficients β_{ij} are determined from the observed data at the modelling stage. By construction, the coefficients have a property that

$$\sum_{i=1}^{I} \beta_{ij} = 1, j = \overline{1, N_{inj}}. \qquad \text{(Equation 6.2)}$$

This ensures that the rate of the total injected water is equal to the rate of the total produced liquid [1, 2]:

$$\sum_{j=1}^{I} i_i = \sum_{i=1}^{P} q_j. \qquad \text{(Equation 6.3)}$$

The purpose of water allocation management is to find such injection rates (i_i), $j = \overline{1, I}$, that maximise the total oil production rate from the reservoir.

If the water cut of the well j is constant, the oil production rate from this well is expressed in terms of the injection rates as follows:

$$q_{o,i} = (1 - WC_i)q_i = (1 - WC_i)\sum_{j=1}^{I} \beta_{ij} i_i. \qquad \text{(Equation 6.4)}$$

Then the objective function can be rewritten in terms of the injection rates as

$$\sum_{i=1}^{P} q_{o,i} = \sum_{i=1}^{P}\left[(1-WC_i)\sum_{j=1}^{I}\beta_{ij} i_i\right] = \sum_{j=1}^{I}\left[\sum_{i=1}^{P}(1-WC_i)\beta_{ij}\right] i_j \rightarrow \text{max.}$$

(Equation 6.5)

There are also constraints. One constraint states that the total injection rate is equal to a certain value i_i^t:

$$\sum_{i=1}^{I} i_i = i_i^t \qquad \text{(Equation 6.6)}$$

There are also two sets of constrains involving the maximum daily amounts of injected water and produced liquid:

$$i_i \le i_i^{\max},\ j = \overline{1, I} \qquad \text{(Equation 6.7)}$$

And

$$\sum_{i=1}^{I} \beta_{ij} i_i \le q_j^{\max},\ i = \overline{1, P} \qquad \text{(Equation 6.8)}$$

We need to ensure that i_i is equal or greater than zero i.e. negative values are not allowed. Thus:

$$i_i \geq 0, i = \overline{1, I}, \tag{Equation 6.9}$$

We can thus formulate the following linear programming (*LP*) problem for the balanced *MLR* in terms of the injection rates:

$$\sum_{i=1}^{I}\left[\sum_{j=1}^{P}(1 - WC_i)\beta_{ij}\right]i_i \rightarrow \max \tag{Equation 6.10}$$

Subject to:

$$\sum_{j=1}^{I} i_i = i_i^t$$

$$\sum_{i=1}^{I}\beta_{ij}i_i \leq q_j^{\max},\ i = \overline{1, P}$$

$$0 \leq i_i \leq i_i^{\max}\ j = \overline{1, I} \tag{Equation 6.11}$$

Let us rewrite the *LP* problem in the vector notation, because this is more compact and more convenient to code in Matlab or Excel. Firstly, denote the rates and the water cuts as column vectors

$$\mathbf{i}_i = \begin{pmatrix} i_{i,1} \\ \vdots \\ i_{i,I} \end{pmatrix},\ i_i^{\max} = \begin{pmatrix} i_i^{\max}{}_{,1} \\ \vdots \\ \overline{i_i^{\max}}{}_{,I} \end{pmatrix}, \mathbf{q}_j = \begin{pmatrix} q_{j,1} \\ \vdots \\ q_{l,P} \end{pmatrix},\ q_j^{\max} = \begin{pmatrix} \overline{q_j^{\max}}{}_{,1} \\ \vdots \\ q_j^{\max}{}_{,P} \end{pmatrix}$$

$$\mathbf{q}_o = \begin{pmatrix} q_{o,1} \\ \vdots \\ q_{o,P} \end{pmatrix},\ \mathbf{w} = \begin{pmatrix} WC_1 \\ \vdots \\ WC_P \end{pmatrix}$$

and the coefficients of the *MLR* model as the $P \times I$ matrix:

$$\mathbf{B} = \begin{pmatrix} \beta_{11} & \cdots & \beta_{1,I} \\ \vdots & \ddots & \vdots \\ \beta_{P1} & \cdots & \beta_{P,I} \end{pmatrix}$$

Secondly, remember some basic linear algebra notation. The vector $\mathbf{1}_n$ is the column n-vector whose entries are all ones, the matrix $\boldsymbol{I}$ is the identity matrix of the required dimensionality (which is clear from the context), $diag(\boldsymbol{x})$ is a matrix with the elements of the n-vector $\mathbf{x}$ on the main diagonal and zeroes elsewhere:

$$\mathbf{1}_n = \Big(\underbrace{1,\dots,1}_{n}\Big)^{\mathrm{T}},\ \mathbf{I} = \begin{pmatrix} 1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & 1 \end{pmatrix},\ \mathrm{diag}(\mathbf{x}) = \begin{pmatrix} x_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & x_n \end{pmatrix}$$

Then

$$\boldsymbol{q}_o = \left(\boldsymbol{I} - diag\left(\boldsymbol{w}\right)\right)\boldsymbol{q}_j = \left(\boldsymbol{I} - diag\left(\boldsymbol{w}\right)\right)\boldsymbol{Bi}_i \qquad \text{(Equation 6.12)}$$

and the objective function takes the form:

$$\mathbf{1}_P{}^{\mathrm{T}}\mathbf{q}_o = \left[\mathbf{1}_P{}^{\mathrm{T}}\left(\mathbf{I} - \mathrm{diag}(\mathbf{w})\right)\mathbf{B}\right]\mathbf{iq}_i \rightarrow \max \qquad \text{(Equation 6.13)}$$

The Equation 6.6 can be rewritten as

$$\mathbf{1}_I{}^{\mathrm{T}}\mathbf{i}_i = i_i^t$$

and Equation 6.7 as

$$\mathbf{Bi}_i \leq q_j^t \qquad \text{(Equation 6.14)}$$

Then the *LP* problem in terms of the injection rates is reformulated as follows:

$$\left[\mathbf{1}_I{}^{\mathrm{T}}\left(\mathbf{I} - \mathrm{diag}(\mathbf{w})\right)\mathbf{B}\right]\mathbf{i}_i \rightarrow \max \qquad \text{(Equation 6.15)}$$

Subject to:

$$\mathbf{1}_I{}^{\mathrm{T}}\mathbf{i}_i = i_i^t \qquad \text{(Equation 6.16)}$$

$$\mathbf{Bi}_i \leq q_j^{\max} \qquad \text{(Equation 6.17)}$$

$0 \leq \mathbf{i}_i \leq i_i^{\max}$ (Equation 6.18)

This problem can be solved using Matlab or Excel (if the number of the injection wells is relatively small).

6.3 *CRM* Review

The *CRM* at the (discrete) time step $t_n,\ n \geq 0$, takes into account the injection rates and change in bottom hole pressures *(BHP)* at the previous time steps as well as the primary production (Yousef et al., 2005; Sayarpour et al., 2009) [3, 4]. Let the liquid production rate q_j from the well *j* be described by the *CRM*:

$$q_j(t_n) = q_j(t_0)e^{\frac{-(t_n-t_0)}{\tau_j}} + \sum_{k=1}^{n}\left[\left(\sum_{i=1}^{I} f_{ij} i_i^k - J_j \tau_j \frac{\Delta P_{wf,j}^{(k)}}{\Delta t_k}\right)\left(1-e^{\frac{-\Delta t_k}{\tau_j}}\right)e^{\frac{-(t_n-t_k)}{\tau_j}}\right] \quad \text{(Equation 6.19)}$$

Where I_{ij}^k and $\Delta P_{wf,j}^{(k)}$ represent the injection rate of injector *i* and the changes in BHP of producer *j* during time interval t_{k-1} to t_k respectively.; Δt_k is the length of the time step k; τ_{ij}, f_{ij} and J_j are the time constant, the fraction of injection rate contributing in production rate and productivity index associated with the volume between the injector i and the producer *j* pair.

Let us change the summation order in the second term of the Equation 6.19:

$$q_j(t_n) = \sum_{i=1}^{I} q_j(t_0)e^{-\left(\frac{t_n-t_0}{\tau_j}\right)} + \sum_{k=1}^{n}\left\{\sum_{i=1}^{I}\left[\left(f_{ij} i_j^{(k)} - J_j \tau_j \frac{\Delta P_{wf,i}^{(k)}}{\Delta t_k}\right)\left(1-e^{-\frac{\Delta t_k}{\tau_j}}\right)e^{-\left(\frac{t_n-t_k}{\tau_j}\right)}\right]\right\}$$

(Equation 6.20)

and split it in three parts:

$$q_j(t_n) = \sum_{i=1}^{I} q_j(t_0) e^{-\left(\frac{t_n - t_0}{\tau_{ij}}\right)} + \sum_{k=1}^{n-1} \left\{ \sum_{i=1}^{I} \left[\left(f_{ij} i_i^{(k)} - J_j \tau_j \frac{\Delta P_{wf,i}^{(k)}}{\sum t_k} \right) \left(1 - e^{-\frac{\Delta t_k}{\tau_j}} \right) e^{-\left(\frac{t_n - t_k}{\tau_j}\right)} \right] \right\}$$

$$- \frac{\Delta P_{wf,i}^{(n)}}{\Delta t_n} \sum_{i=1}^{I} \left(1 - e^{-\frac{\Delta t_n}{\tau_j}} \right) J_j \tau_j + \sum_{i=1}^{I} \left(1 - e^{-\frac{\Delta t_n}{\tau_{ij}}} \right) f_{ij} i_i^{(n)}$$

(Equation 6.21)

It is easy to see that:

1. $q_j(t_n)$ linearly depends on the $\left\{i_i^{(n)}\right\}_{i=1}^{I}$.
2. The sum of the first three terms is constant at the time step t_n,, and, hence, does not affect the objective function gradient. This means that the gradient depends neither on the initial production nor on the prehistory of injection rates and *BHP*'s.
3. If the time step length Δt_n is the same for all n, then the coefficients at $\left\{q_i^{(n)}\right\}_{i=1}^{I}$ are the same for all the time steps. This means that the objective function gradient is the same all the time and only needs to be calculated once, at the beginning.
4. Any constraints on the production rate depend on the initial production, the prehistory of injection rates and *BHPs*, as well as the current value of *BHP* for each production well (we presume that the *BHP* can be measured).

If the production wells' *BHPs* are constant, Equation 6.21 can be simplified:

$$q_j(t_n) = \sum_{i=1}^{I} q_j(t_0) e^{-\left(\frac{t_n - t_0}{\tau_j}\right)} + \sum_{k=1}^{n-1} \left\{ \sum_{i=1}^{I} \left[\left(1 - e^{-\frac{\Delta t_k}{\tau_j}} \right) e^{-\left(\frac{t_n - t_k}{\tau_{ij}}\right)} f_{ij} i_i^{(k)} \right] \right\} + \sum_{i=1}^{I} \left(1 - e^{-\frac{\Delta t_n}{\tau_j}} \right) f_{ij} i_i^{(n)}$$

(Equation 6.22)

The structure of the model is identical to that of the unbalanced *MLR*: a free term plus a scalar product of a vector of coefficients and the vector of the injection rates to be found. Let us denote

$$\beta_{i0}^{(n)} = \sum_{i=1}^{I} q_j(t_0) e^{-\left(\frac{t_n-t_0}{\tau_j}\right)} + \sum_{k=1}^{n-1}\left\{\sum_{i=1}^{I}\left[\left(f_{ij} i_i^{(k)} - J_j\tau_j \frac{\Delta P_{wf,i}^{(k)}}{\Delta t_k}\right)\left(1 -\right.\right.\right.$$

$e-\Delta tk\tau je-tn-tk\tau j-\Delta Pwf,i(n)\Delta tnj=1II1-e-\Delta tn\tau jJj\tau j$

(Equation 6.23)

$$\beta_{ij}^{(n)} = \left(1 - e^{-\frac{\Delta t_n}{\tau_j}}\right) f_{ij} \qquad \text{(Equation 6.24)}$$

where $i = \overline{1,P}$ and $j = \overline{1,I}$. Let us introduce the following vectors and matrix:

$$\mathbf{i}_i^{(n)} = \begin{pmatrix} i_{i,1}^{(n)} \\ \vdots \\ i_{i,I}^{(n)} \end{pmatrix}, \qquad \mathbf{b}_0^{(n)} = \begin{pmatrix} \beta_{10}^{(n)} \\ \vdots \\ \beta_{P,0}^{(n)} \end{pmatrix}, \qquad \mathbf{B}^{(n)} = \begin{pmatrix} \beta_{11}^{(n)} & \cdots & \beta_{1,I}^{(n)} \\ \vdots & \ddots & \vdots \\ \beta_{P,1}^{(n)} & \cdots & \beta_{P,I}^{(n)} \end{pmatrix}.$$

The water cuts can be taken from the previous time step:

$$\mathbf{w}^{(n-1)} = \begin{pmatrix} WC_1^{(n-1)} \\ \vdots \\ WC_P^{(n-1)} \end{pmatrix}$$

If, as above, $i_i^{\max}$ and $q_j^{\max}$ mean the upper limits of the injection and liquid rates for all time steps, the following *LP* problem can be formulated at each time step t_n:

$$\left[\mathbf{1}_P{}^{\mathrm{T}}\left(\mathbf{I} - \mathrm{diag}\left(\mathbf{w}^{(n-1)}\mathbf{w}\right)\right)\mathbf{B}^{(n)}\right]\mathbf{i}_i^{(n)} \to \max$$

(Equation 6.25)

subject to:

$$\mathbf{1}_{N_{inj}}{}^{\mathrm{T}}\mathbf{i}_i^{(n)} \leq i_i^t \qquad \text{(Equation 6.26)}$$

$$\mathbf{B}^{(n)}\mathbf{i}_i^{(n)} \leq q_j^{\max} - \mathbf{b}_0^{(n)} \qquad \text{(Equation 6.27)}$$

$$0 \leq \mathbf{i}_i^{(n)} \leq q_j^{\max} \qquad \text{(Equation 6.28)}$$

This *LP* problem should be solved at each time step. If the upper rate limits and the total amount of water i_i^t change in time, this can be easily accommodated in the *LP* problem.

6.4 Linear Programming (*LP*)

As discussed in the previous sections, *MLR* and *CRM* are both linear, so their constraints and objective functions are linear as well; therefore, Linear Programming can be used as an optimiser to combine with these two estimation methods in order to optimise the water allocation.

Linear programming is the name of a branch of applied mathematics that deals with solving optimization problems of a particular form. Linear programming problems consist of a linear cost function (consisting of a certain number of variables) which is to be minimized or maximized, subject to a certain number of constraints [5]. The constraints are linear inequalities of the variables used in the cost function. The cost function is also sometimes called the objective function. Linear programming is closely related to linear algebra; the most noticeable difference is that linear programming often uses inequalities in the problem statement rather than equalities [6]. *LP* has been used by petroleum engineers for range of optimising problems in the petroleum industry [6-10].

6.5 Case study

The example used in this chapter is the same model studied in previous chapters.

1. *MLR* and *CRM* coefficients are calculated based on 20 years production and injection history. This was already done in Chapter 3.
2. Developed *MLR* and *CRM* models are used for forecasting future oil production.
3. *LP* is applied periodically each 5 years for determining optimum injection rates for each injector in order to maximise daily oil production rate for the next 20 years of production. *LP* is solved by a third party product; in this case, Excel Solver, which was sufficient for solving the *LP* in this study.
4. Minimum and maximum limits are applied for injection rates. The total injection rate is defined to be the same as the total rate of production (*VR=1*).

The results of calculated allocation factor for injectors are shown in Figures 6.1 to 6.3, for both cases (in both cases the allocation results were the same).

As can be seen, the overall waterflooding performance improved: there is an increase in oil production rate and decrease in water production. However, in both cases (*MLR* and *CRM*), after changing the injection rate for the first 5 years, there was a sudden change in water cut of producer 2 (Figure 6.4). This caused significant change in the calculated optimum allocation factor for each period (Figure 6.5). As a result, we can see fluctuation in the production rate (Figure 6.1). The same problem happened when the water cut and the connectivity measurement were used together for water allocation management, as reported in the previous chapter. In order to solve this, we changed the objective function from maximising daily oil production to maximising cumulative oil production.

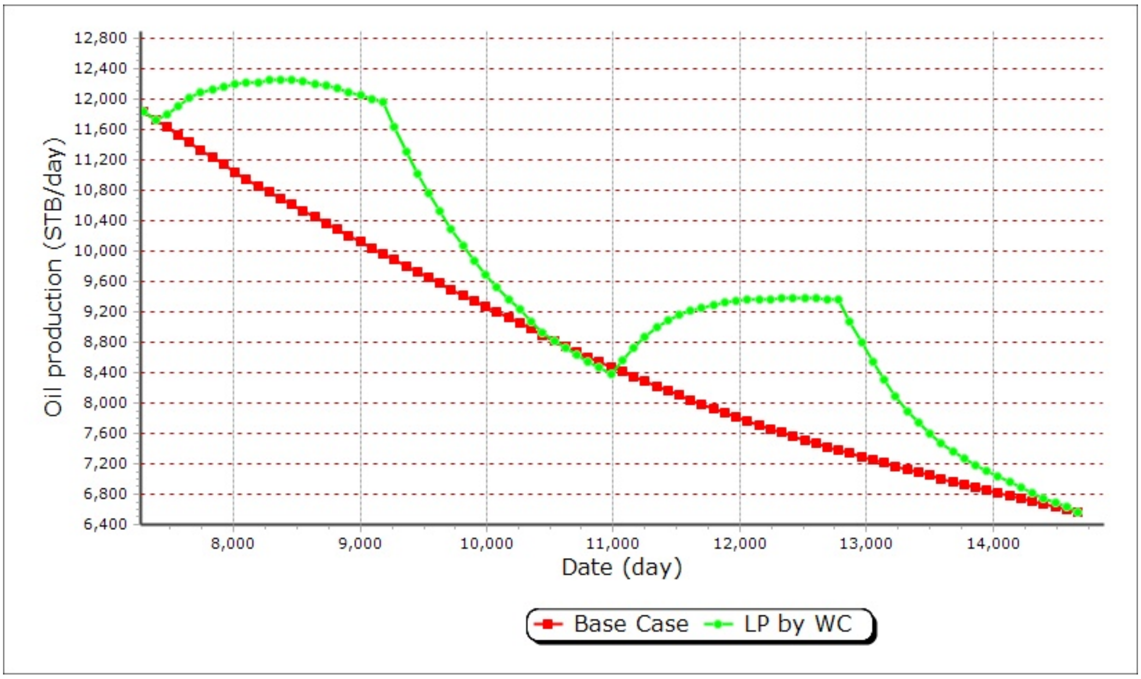


Figure 6. 1: Oil production rate for base case and WAM by LP.

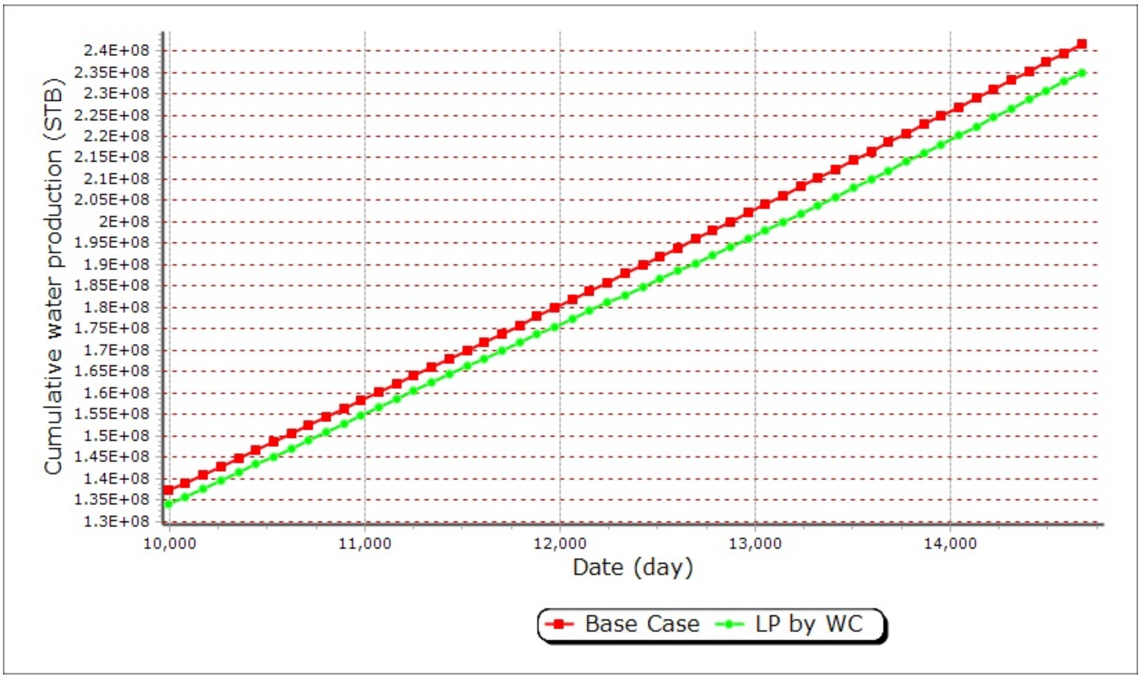


Figure 6. 2: Cumulative water production for base case scenario and WAM by LP.

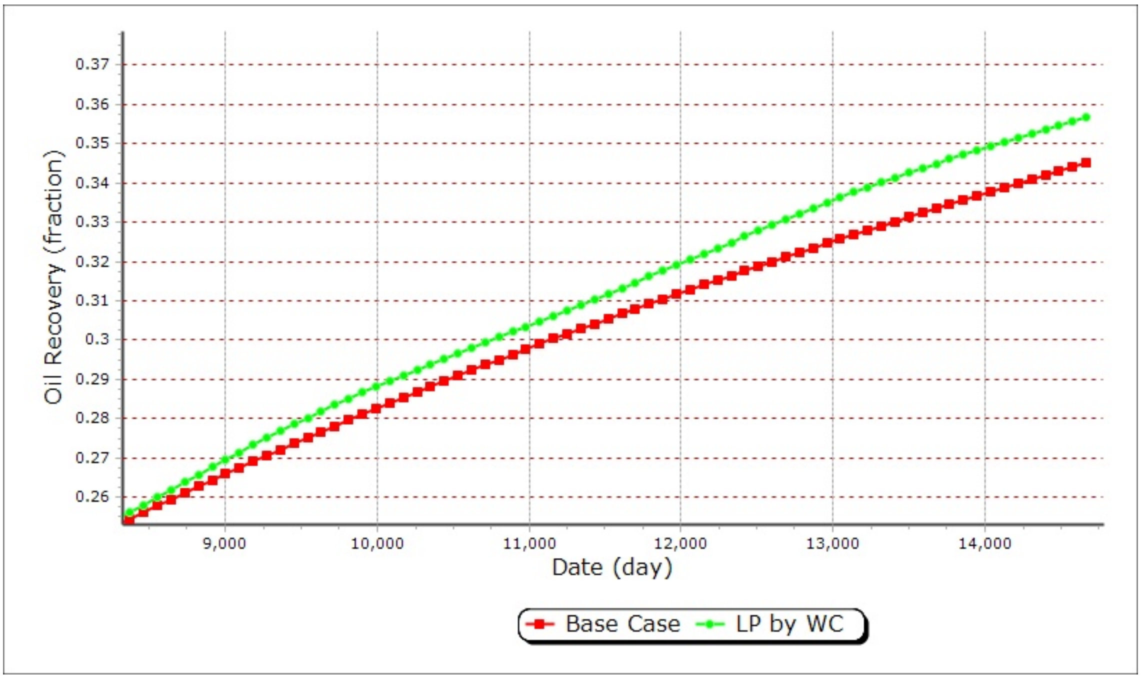


Figure 6. 3: Oil recovery versus time for base case scenario and WAM by LP.

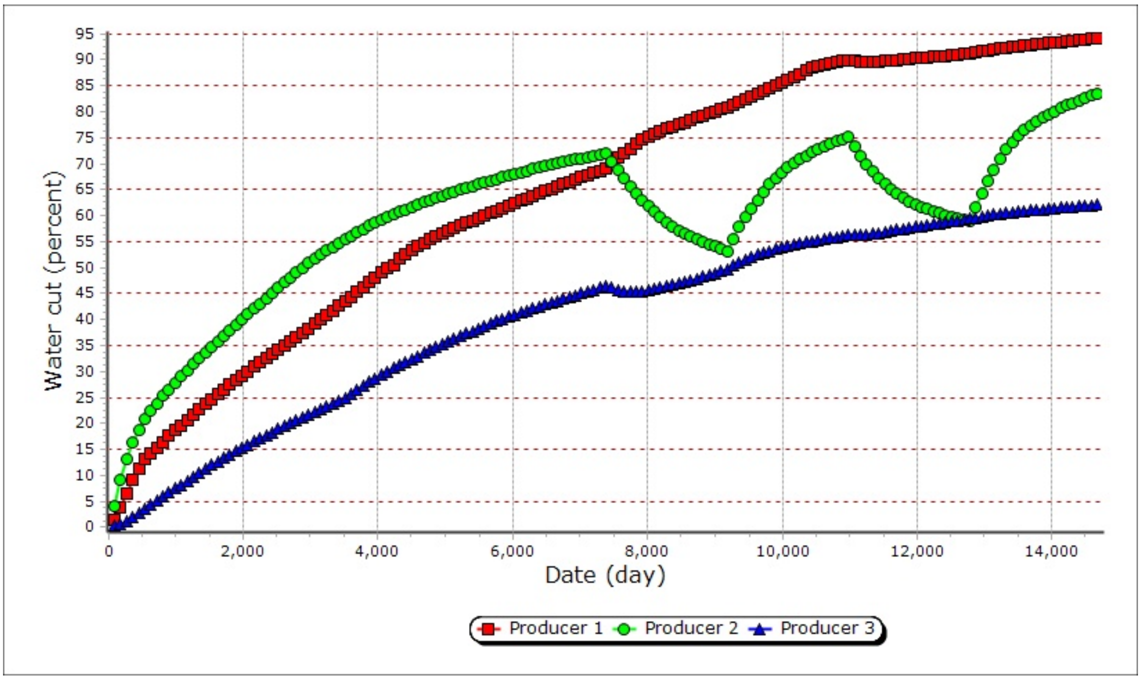


Figure 6. 4: Water cut development of the 3 production wells for WAM scenario by LP.

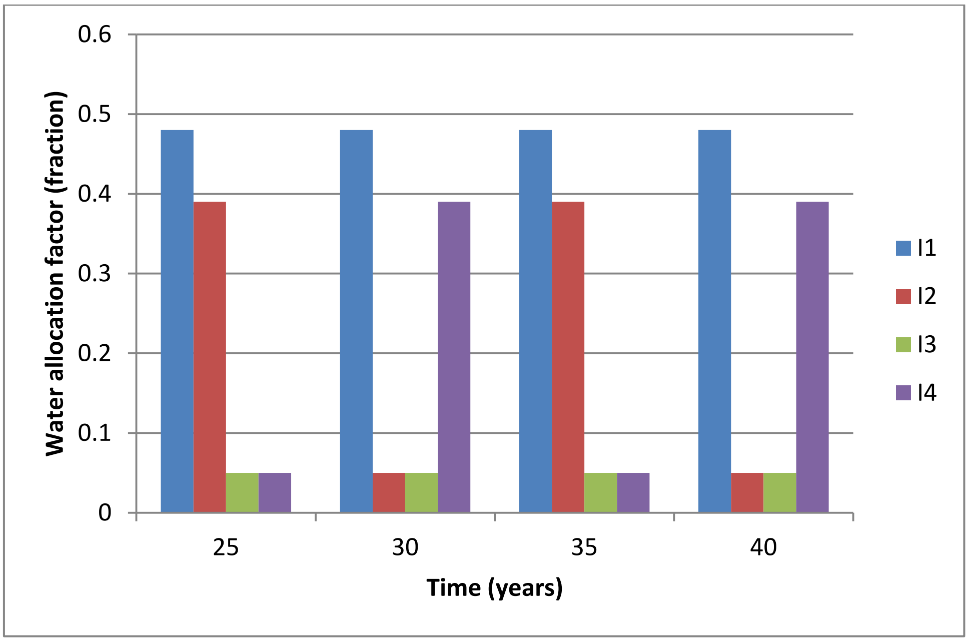


Figure 6. 5: Calculated water allocation factor at the end of each 5-year period.

The next approach to be studied is based on maximising the cumulative oil production.

Cumulative liquid production from producer *i* is:

$$Q_{l,i} = q_{l,i} \times t \quad \text{(Equation 6.29)}$$

in which t is the production period. The cumulative oil production from producer i is:

$$Q_{o,i} = COC_i \times Q_{l,i} \qquad \text{(Equation 6.30)}$$

In equation 6.27 COC_i is the cumulative oil production cut of producer i, which is defined by:

$$COC_j = \frac{Cumulative\ Oil\ production\ of\ producer\ j}{Cumulative\ Liquid\ production\ of\ producer\ j} \qquad \text{(Equation 6.31)}$$

The objective function is to maximise the total cumulative oil production of the reservoir:

$$Q_{o,res} = \sum_{i=1}^{N_{prod}} Q_{o,i} \rightarrow Max \qquad \text{(Equation 6.32)}$$

Since change in COR is not significant and it will respect the previous production performance of the producer in comparing with water cut, there is no fluctuation in calculated optimum allocation factor (Figure 6.6).

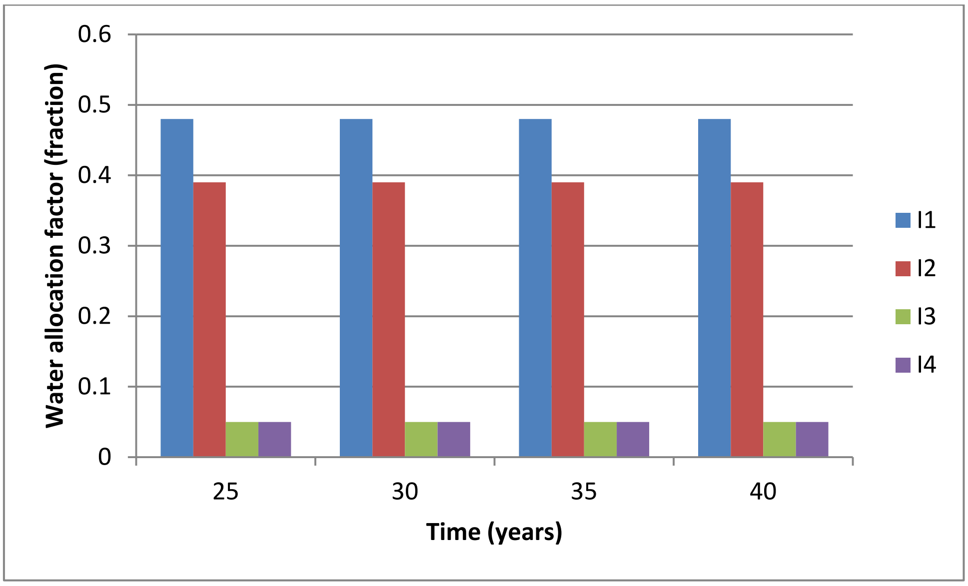
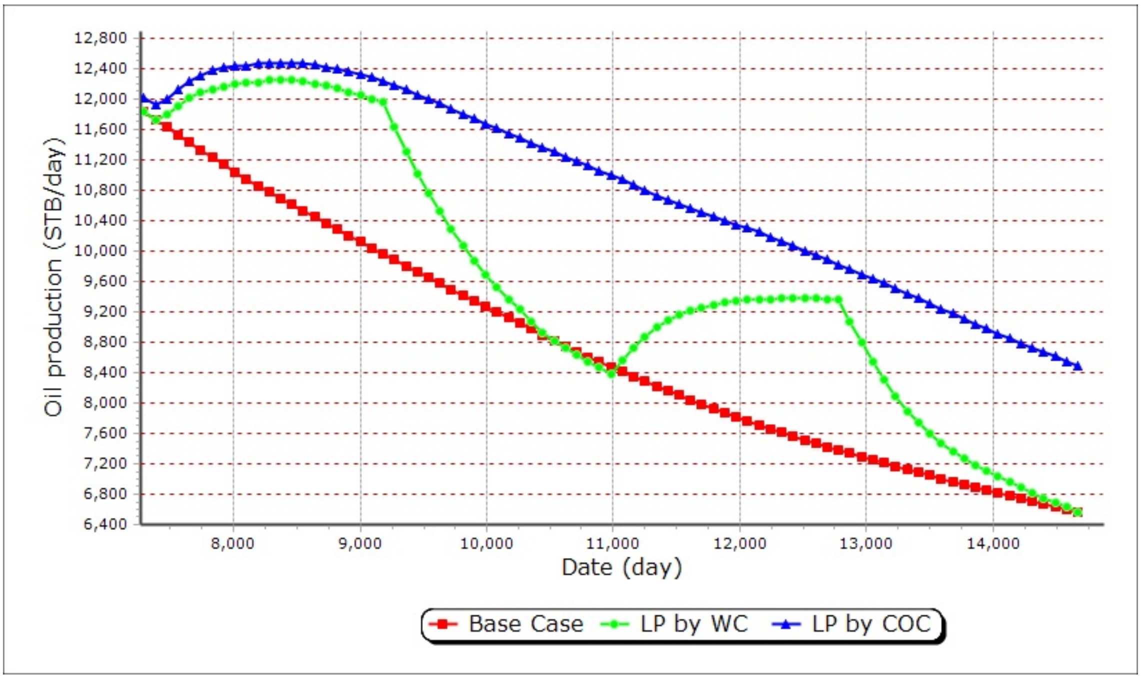


Figure 6. 6: Calculated allocation factor for each period based on optimizing cumulative oil production as objective function.



6. 1: Comparison of improvement in oil production rate between base case, WAM by LP and WC and LP and COC.

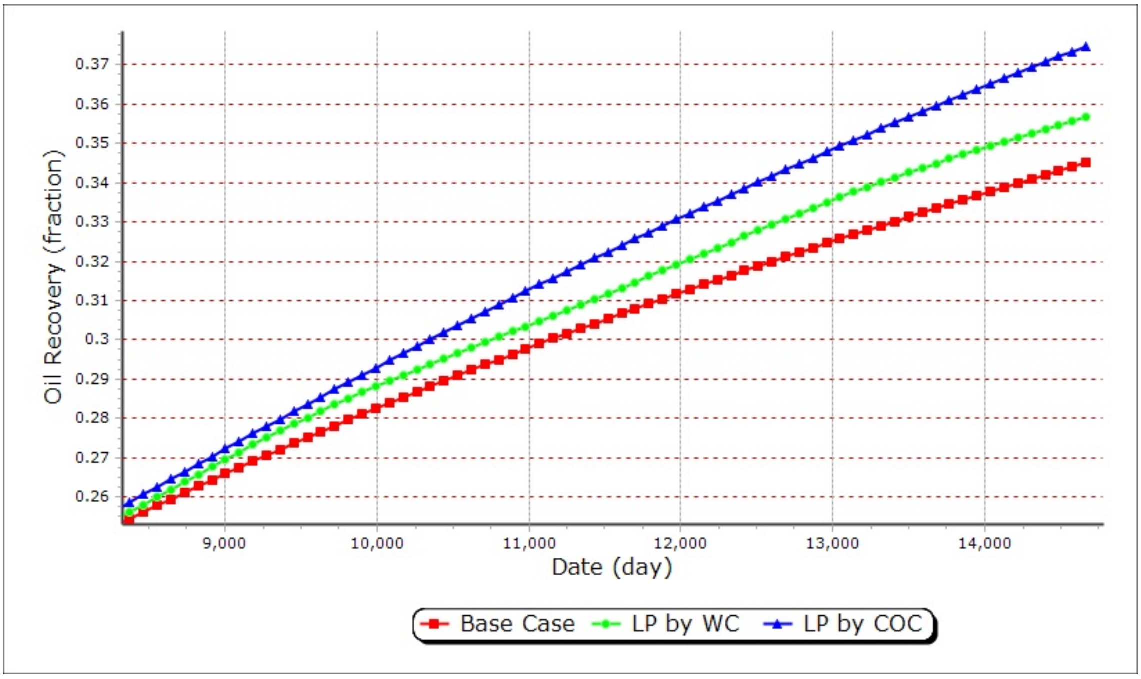


Figure 6. 7: Oil recovery versus time for base case, WAM by LP and WC and LP and COC.

As a result, stable daily oil production is obtained from this method and the cumulative oil production has been improved (Figures 6.7 to 6.11).

Application of *LP* combined with *MLR* and the *CRM* model improves waterflooding efficiency in terms of more oil recovery and less water production. *LP* problems are sensitive to constraints (including technological limits). Incompatible constraints result in infeasible problems (i.e., with no feasible solution).Better results are obtained from optimisation based on *COR*, compared with those based on *WC*, for both *CRM* and *MLR* cases.

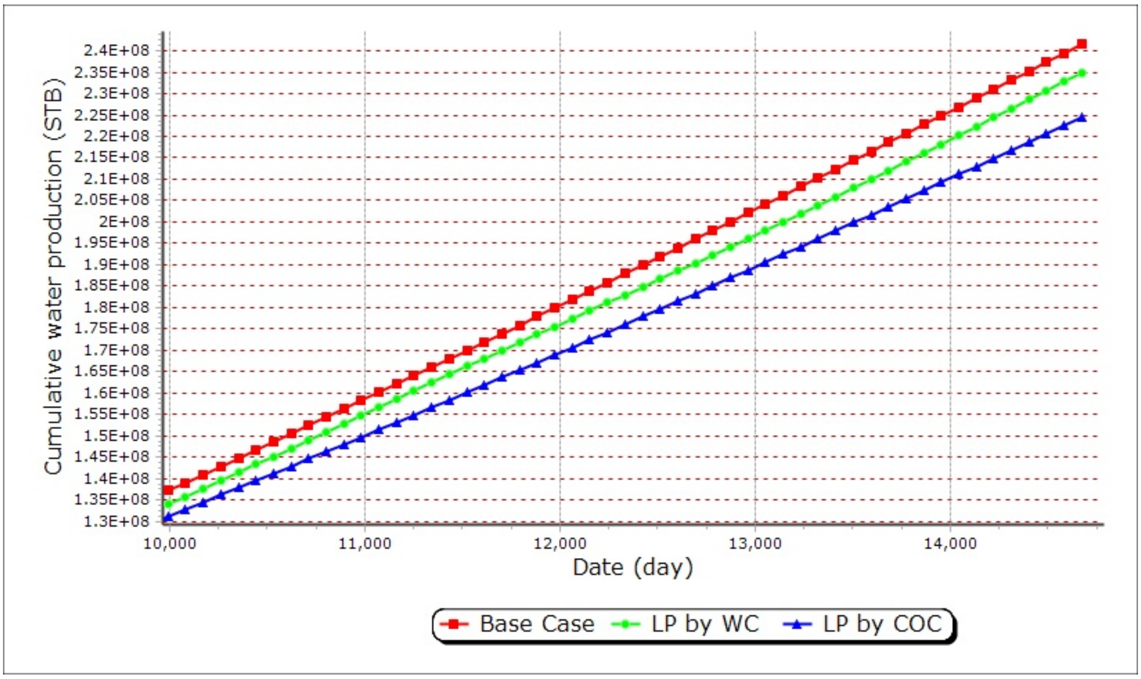


Figure 6. 8: Cumulative water production versus time for base case, WAM by LP and WC and LP and COC.

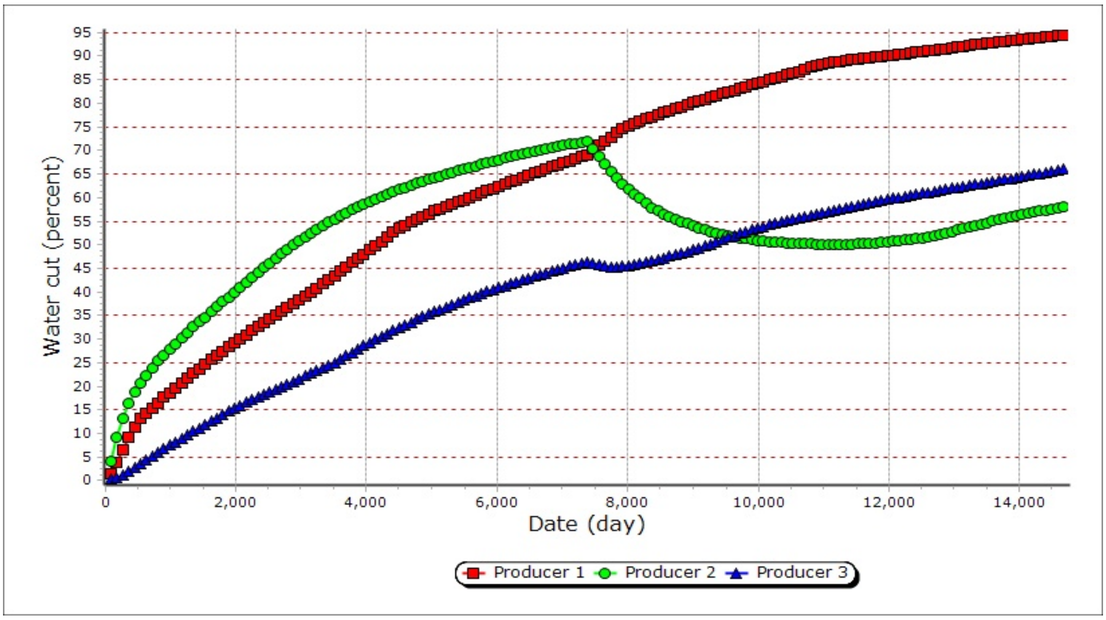


Figure 6. 9: Change in producer's water cut in case of WAM with LP and COC.

6.6 New optimisation using Genetic Algorithm (*GA*) and reservoir simulator

There are two important issues concerning the combination of *LP* with *MLR* or *CRM* to optimise water allocation. Firstly, *CRM* or *MLR* involve a very simplified representation of the reservoir, compared with a reservoir simulator. Secondly, *LP* is a gradient based optimiser. This kind of optimisation method is very good at finding local optima , but they may fail to obtain the global optimum. A new optimisation is proposed, to overcome these limitations; a combination of genetic algorithm and reservoir simulator.

In general, this new approach is the same as the previous one, but the production rate estimators (*MLR* and *CRM* techniques) are replaced by the reservoir simulator and the optimization algorithm is changed from *LP* to *GA*.

Genetic Algorithms (*GAs*) are adaptive heuristic search algorithms premised on the evolutionary ideas of natural selection and genetics [11]. The basic concept of *GAs* is designed to simulate processes in a natural system necessary for evolution, specifically those that follow the principles first laid down by Charles Darwin of survival of the fittest. As such they represent an intelligent exploitation of a random search within a defined search space to solve a problem. There have been several application of *GAs* in petroleum engineering [10-13].

Fortunately, the Reveal reservoir simulator has a feature that allows us to control the simulation by defining a batch file. A script is developed that can change the control variable in Reveal, run the simulation and read the results and calculate the objective function. This script is also equipped with an in-house *GA* optimiser that maximises the objective function by changing defined control variables.

The objective function was maximizing the oil recovery by changing the injection rate of the injection wells. Producers were producing with constant liquid production rate. Injection was controlled by voidage replacement. There were maximum and minimum limits for injection rates. As in the previous work the simulation is run for 20 years without allocation management and then a *GA* is used to find the optimum injection rate for the next 20 years.

The optimum solution was obtained after nearly 50 iterations, with 10 simulations run in each iteration (Figure 6.11and 6.12). The sequential run time was 16 hours with one

CPU; of course it could be reduced by parallel calculation. Figures 6.12 to 6.14 compare the results of these two approaches for water allocation management.

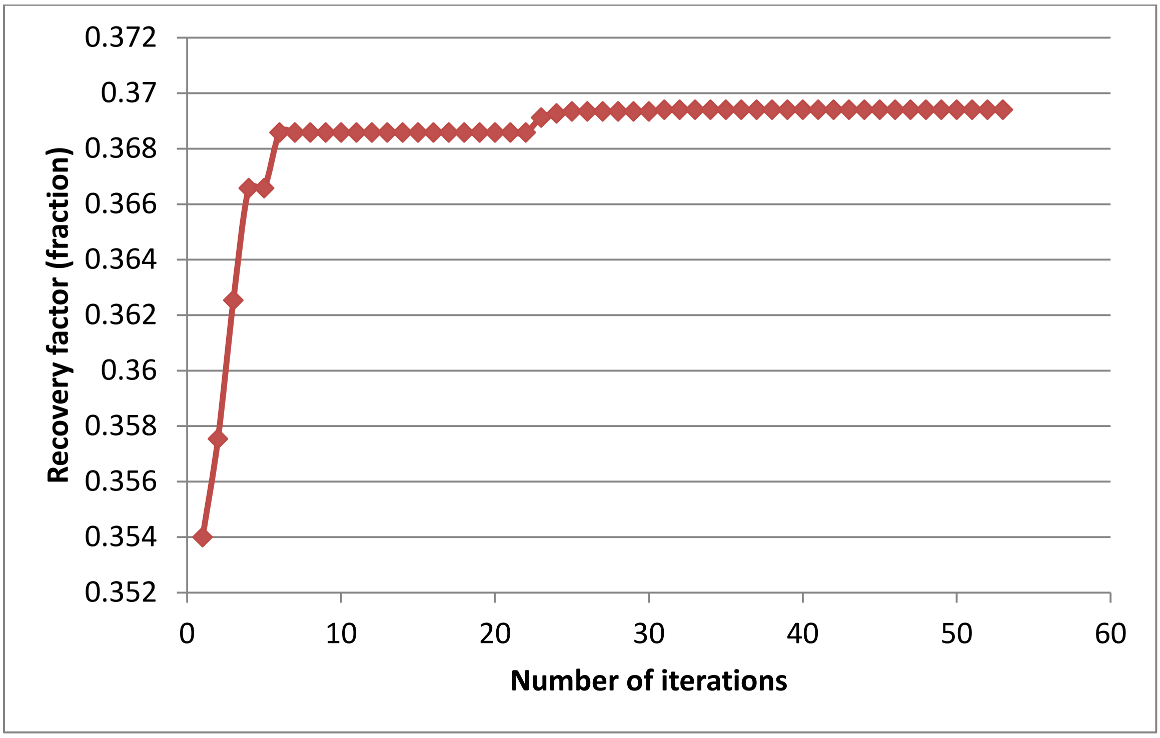


Figure 6. 10: GA solution development over 50 iterations.

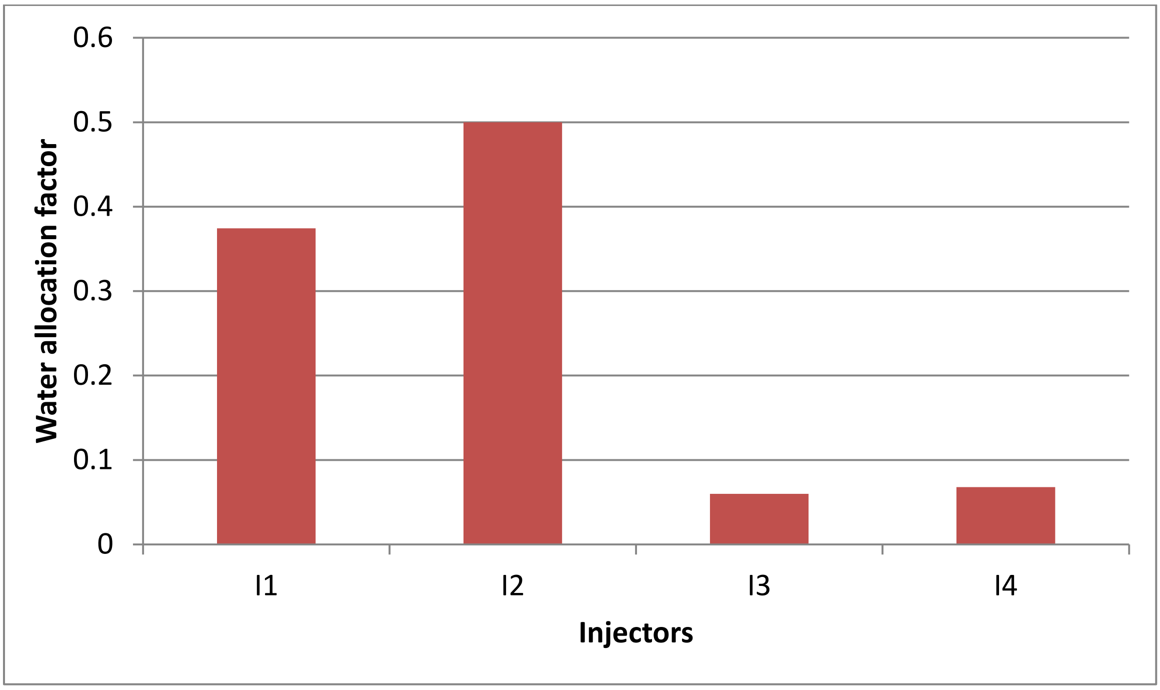


Figure 6. 11: Optimum water allocation factor obtained from GA for each injection well.

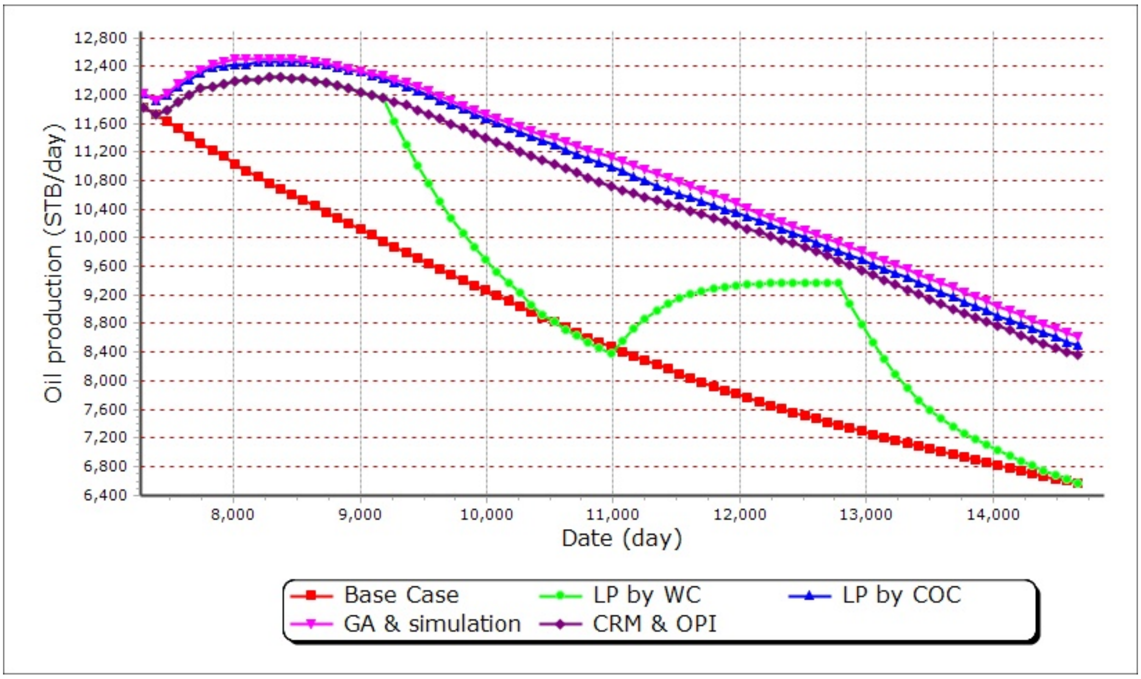


Figure 6. 12: Plot of oil production rate versus time for different WAM scenarios.

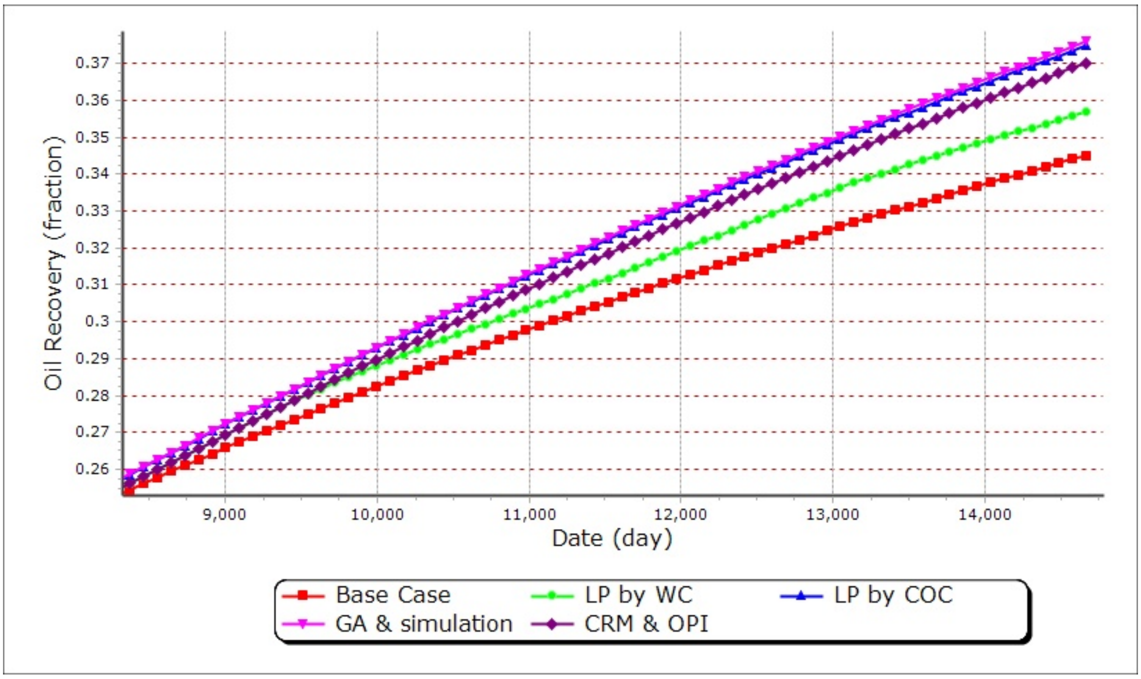


Figure 5. 36: Plot of oil recovery rate versus time for different WAM scenarios.

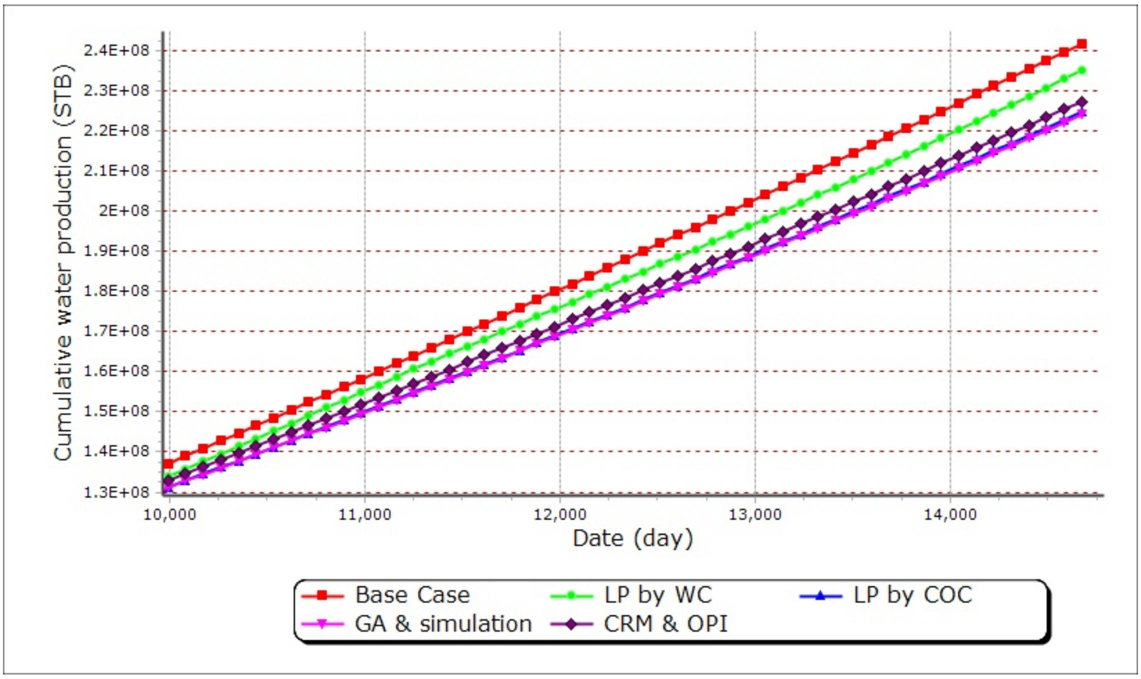


Figure 5. 37: Plot of cumulative water production versus time for different WAM scenarios.

Waterflooding performance was improved in terms of increased oil recovery and less water production by GA-simulation technique. Comparing the results of *OPI-CRM* water allocation management with *LP-CRM* indicates better improvement in water injection performance by *LP-CRM*. This shows that optimisation based on future forecast of production works better than water allocation management based on previous history of production.

Comparison of water allocation optimisation with GA-simulation and *LP-CRM* with *COC* shows that only small improvement obtained from GA-simulation. But the main disadvantage of GA-simulation was the substantial increase in computation time. Therefore in very large and more complex reservoir *LP-CRM* is an effective tool for water allocation management. It is easy to apply and very fast to determine the optimum solution. And if sophisticated reservoir simulators with powerful computers are available then GA-simulation can be employed to determine optimum water allocation factor. In this case *LP-CRM* can be used to determine the initial guess for GA-simulation technique in order to reduce the computational time.

6.7 References

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Chapter 7– Waterflooding Management: New Case Study

In the previous chapters, new workflows have been developed in order to improve waterflooding performance based on the analysis of the injection and production history. It has been realised that waterflooding management can be performed by: reservoir voidage management (*RVM*), water allocation management (*WAM*) and production allocation management (*PAM*). The proposed techniques were applied to a small reservoir model and the results showed significant improvement in the performance of the water injection.

In this chapter we show how we applied those techniques to a real field reservoir model. This study aims to evaluate the added value of these new methodologies on a more complex reservoir model.

7.1 Model description

The reservoir model has 8 producers supported with 4 injectors. The properties of the model are given in Table 7.1 and Figures 7.1 and 7.2.

Unfortunately there was no information available about the geology and the formation structure of the model.

Table 7. 1: Reservoir model properties

Number of cells	41×80×21
Initial oil in place	7.98×10^{10} STB
Initial reservoir pressure	5692 psia
Temperature	208^{o} F
Compressibility	2×75^{6} 1/psi
Oil density	40 API
Bubble point pressure	2171 psia
Gas oil ratio	520 Scf/STB

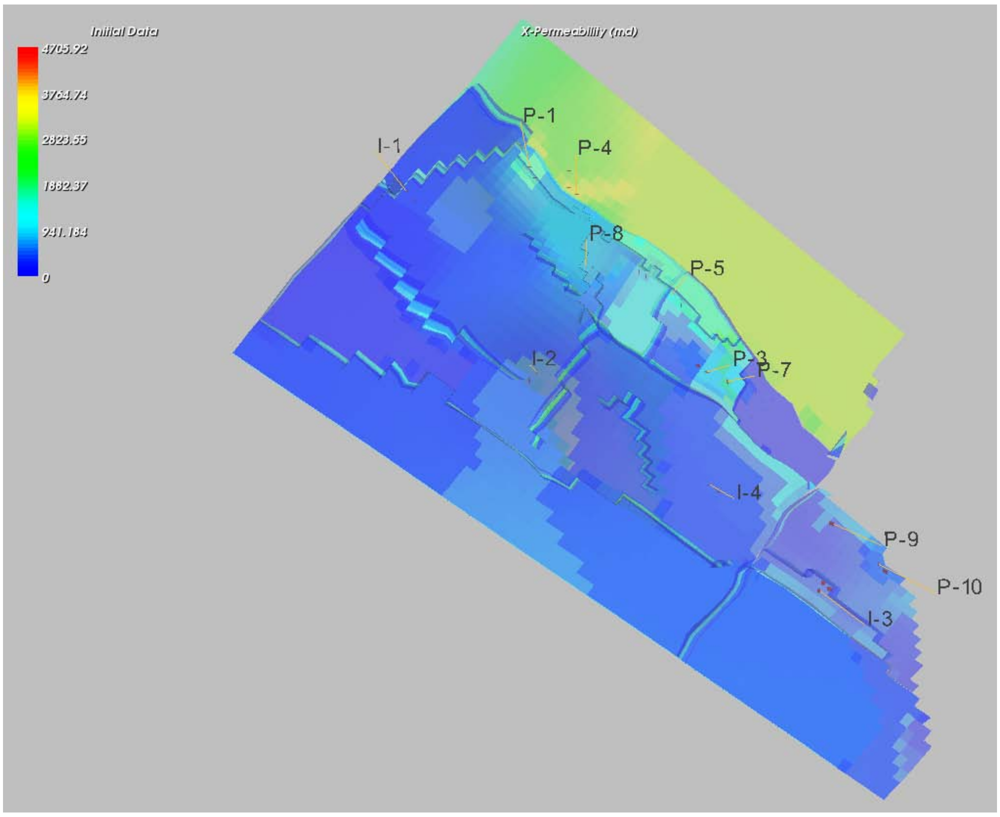

Figure 7. 1: Permeability variation in the reservoir model.

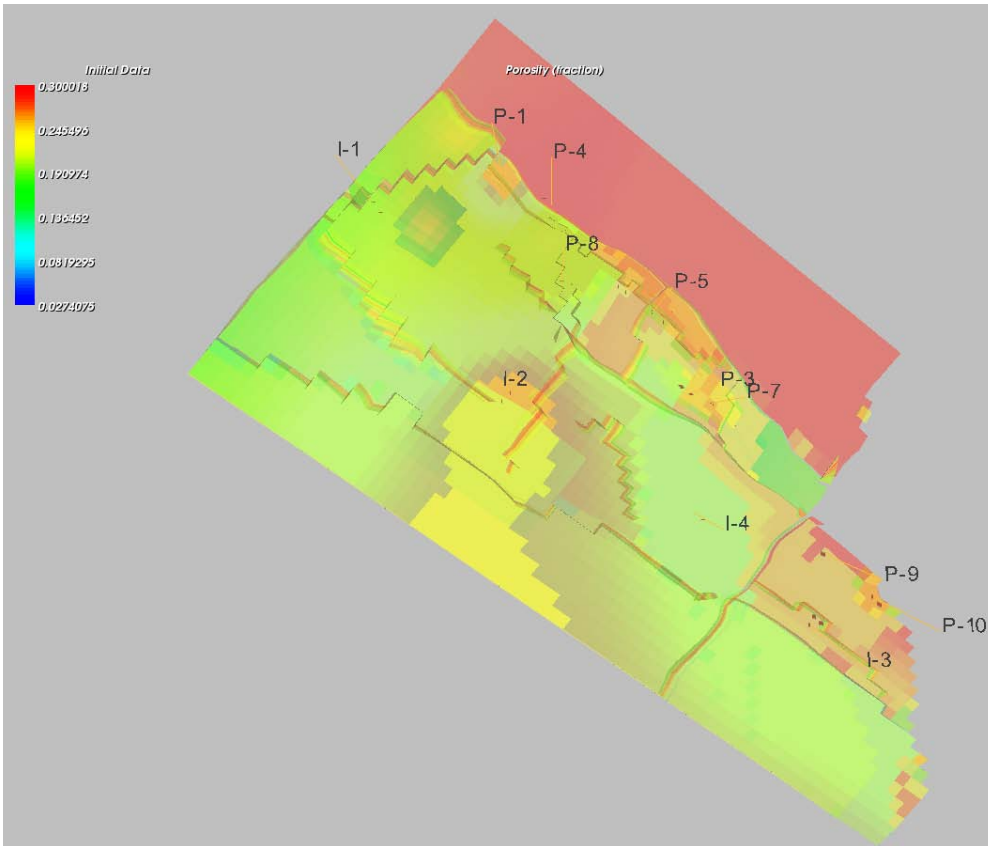

Figure 7. 2: Porosity variation in the reservoir model.

7.2 Reservoir voidage management (*RVM*)

The semi-analytical method developed in Chapter 2 is applied for *RVM*. The base case scenario is run for 30 years with the following injection and production rates schemes: water has been injected with total voidage replacement ratio of 1; this water is equally allocated to the injectors; production is controlled by constant liquid production. Table 7.2 shows the allocated liquid production rate of each production wells. Simulation results showed that water breakthrough will happen after cumulative injection of 25,130,960 STB water.

At first the minimum required reservoir pressure was determined by running a sensitivity analysis on well outflow performance (Figure 7.3). According to this analysis, the minimum required reservoir pressure for this reservoir to support production in producers is 2800 psi.

Table 7. 2: Allocated liquid production rate of each production well for base case scenario.

Production well	Liquid production rate (STB/day)
P1	16000
P10	14000
P3	7000
P4	12000
P5	16000
P7	5000
P8	10000
P9	13000

Equation 2.23 is used to determine the time of the starting pressure maintenance (injection with VR of 1) (t_{pm}). Table 7.4 shows the calculated t_{pm} and the cumulative injected water before reaching the P_{rmin} (Q_{ipm})for each *VR*. Calculated Q_{ipm} for each *VR* is bigger than the cumulative injected water before water breakthrough. Therefore, water breakthrough will occur before the start of pressure maintenance for all of the *VRs*. In order to maximise the breakthrough time we need to inject as little as we can. The proposed voidage replacement plan for this field is to inject with VR 0.1 until the reservoir pressure declines to minimum level (2587 days) then start to inject with VR of 1 for the rest of the production period.

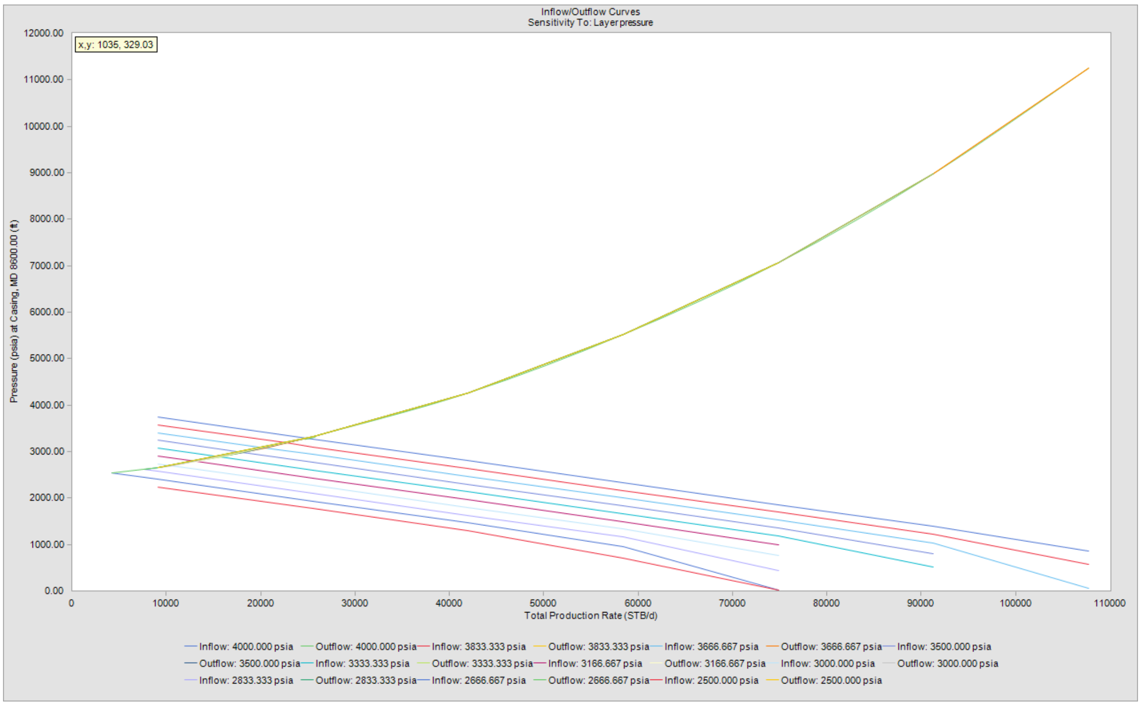

Figure 7.3: Well outflow sensitivity analysis shows that the minimum required reservoir pressure is 2800 psi.

Table 7. 3: Calculated t_{pm} and Q_{ipm} for each VR.

VR	t_{pm}	Q_{ipm}
0.9	23288.82	2971700607
0.8	11644.41	1320755826
0.7	7762.942	770440898
0.6	5822.206	495283435
0.5	4657.765	330188956
0.4	3881.471	220125971
0.3	3326.975	141509553
0.2	2911.103	82547239.1
0.1	2587.647	36687661.8

7.3 Production allocation management *(PAM)*

The downhole production rate of the producers is used to define new production allocation factor for production wells. *PAM* has been done based on the oil relative production ratio (*OPR*) and was applied after starting of the pressure maintenance.

Table 7.4 shows the calculated *OPR* for each of the production wells. The new and old production allocation factors are shown in Figure 7.4.

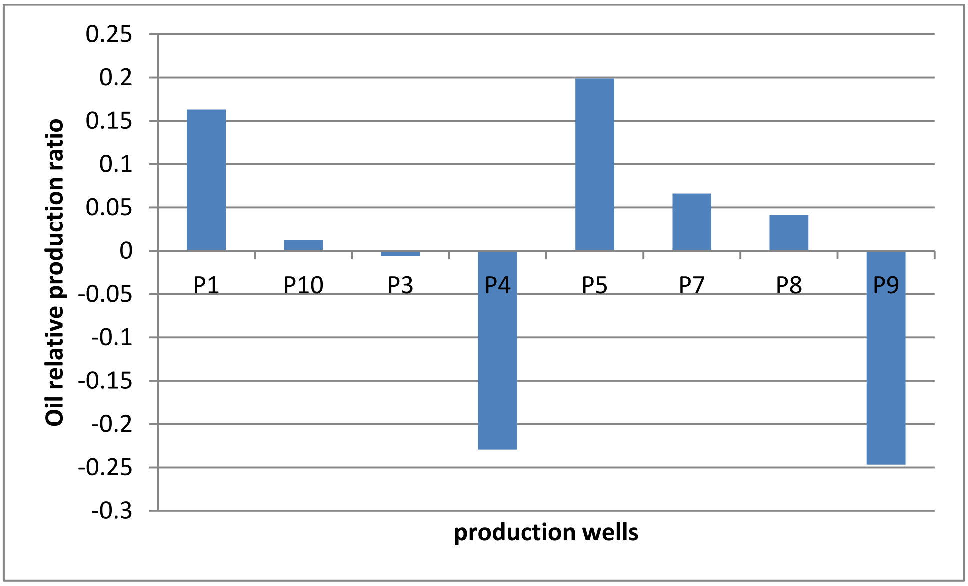

Figure 7. 4: Calculated OPR for all production wells.

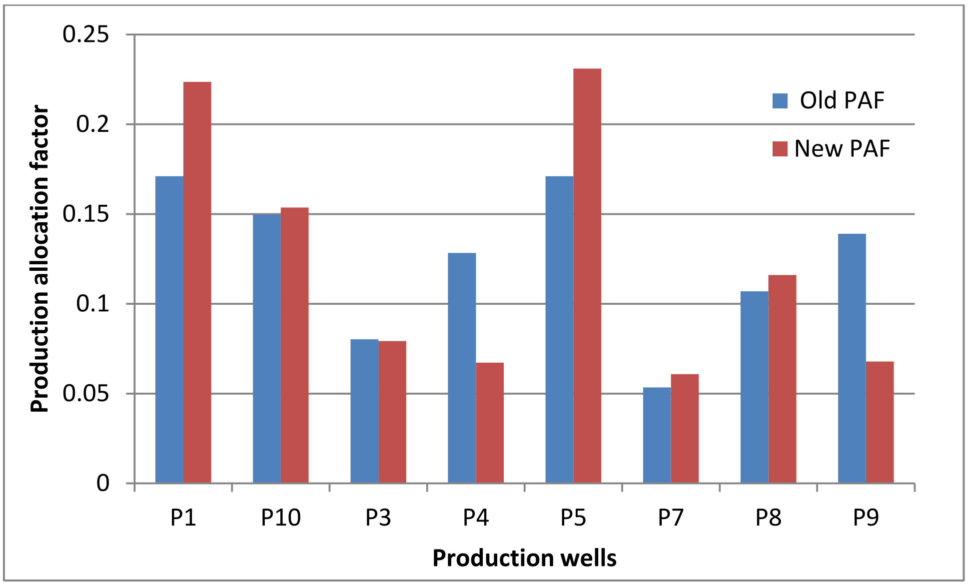

Figure 7.5: Old and new values of PAF for each producer.

7.4 Water allocation management (*WAM*)

WAM was employed by *LP-CRM* method. 15 years of injection and production history were used to determine the coefficients of *CRM* for each individual production well. The following Figures 7.6 to 7.9 show the *CRM* estimated production rate versus the original production rate of some of the producers.

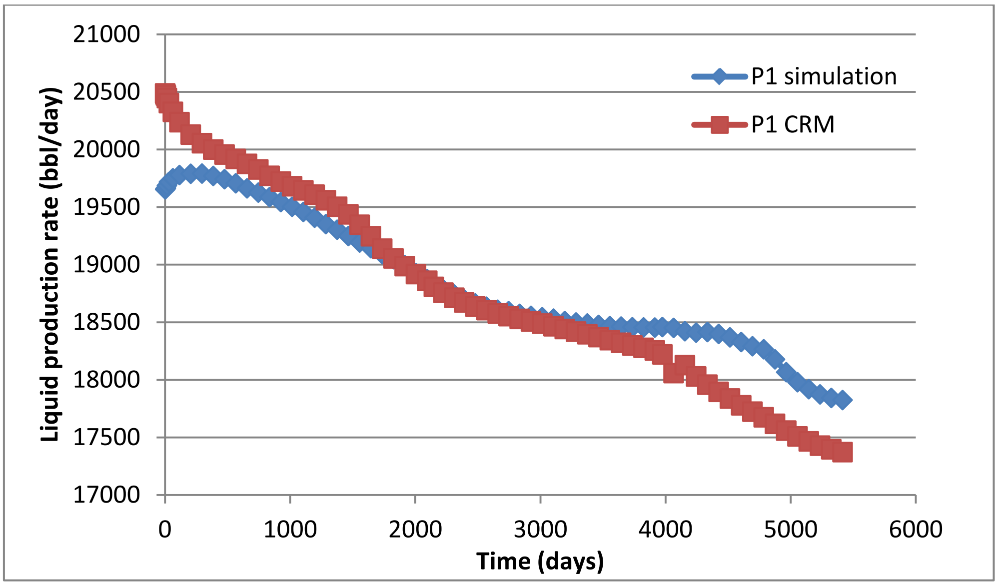

Figure 7. 6: Calculated production rate from CRM versus production history of Producer 1 from reservoir model.

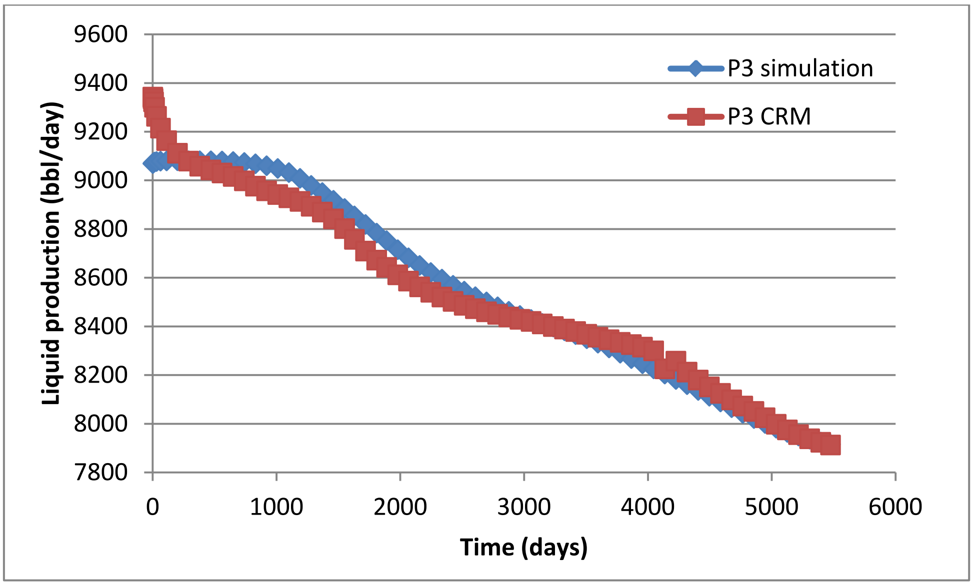

Figure 7. 7: Calculated production rate from CRM versus production history of Producer 3 from reservoir model.

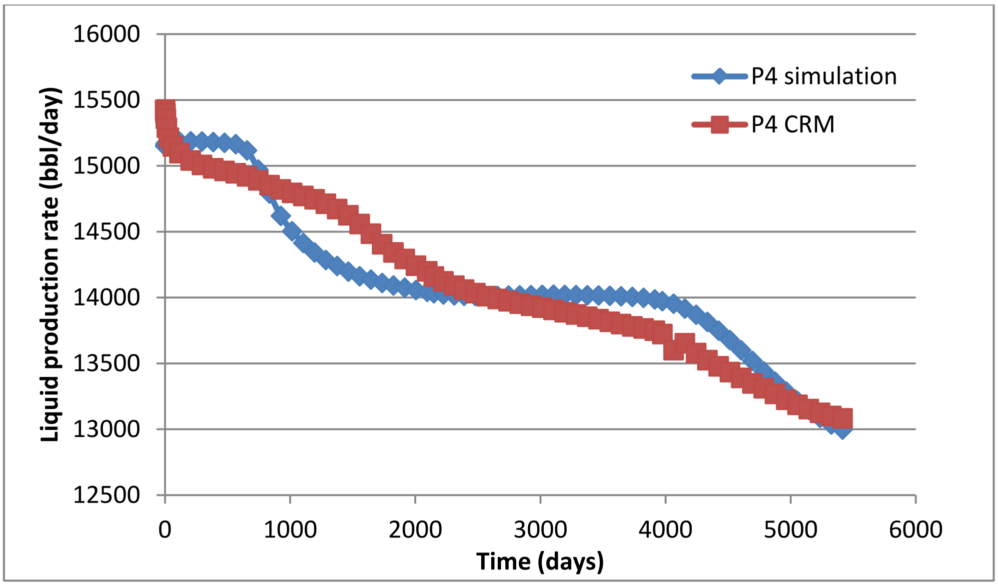

Figure 7. 8: Calculated production rate from CRM versus production history of Producer 4 from reservoir model.

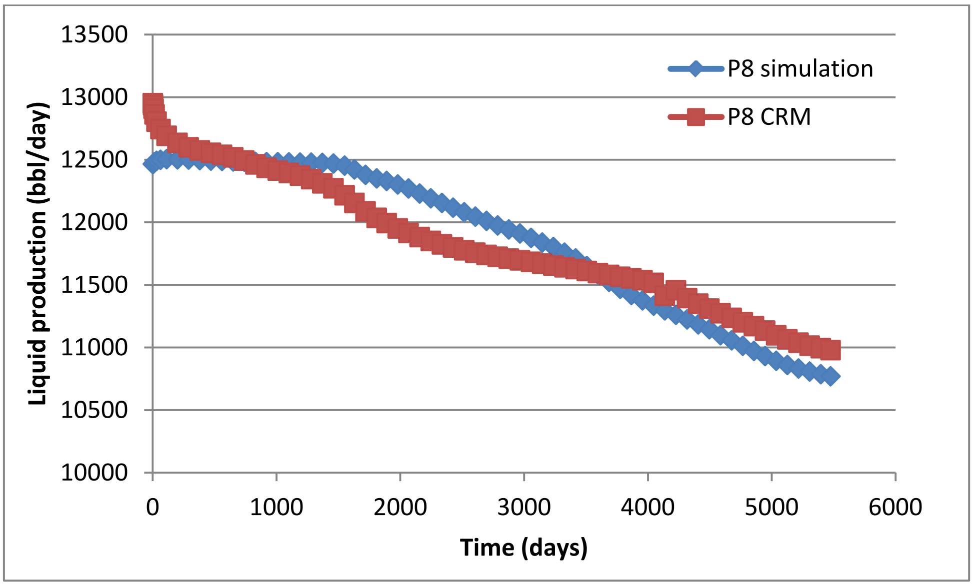

Figure 7.9: Calculated production rate from CRM versus production history of Producer 8 from reservoir model.

Figure 7.10 shows the calculated coefficients of *CRM* for each production well. *LP* was then performed to determine the optimum water allocation factor for the injection wells. The objective function was to maximise the cumulative oil production. The new allocation injection allocation factor is shown in Figure 7.11.

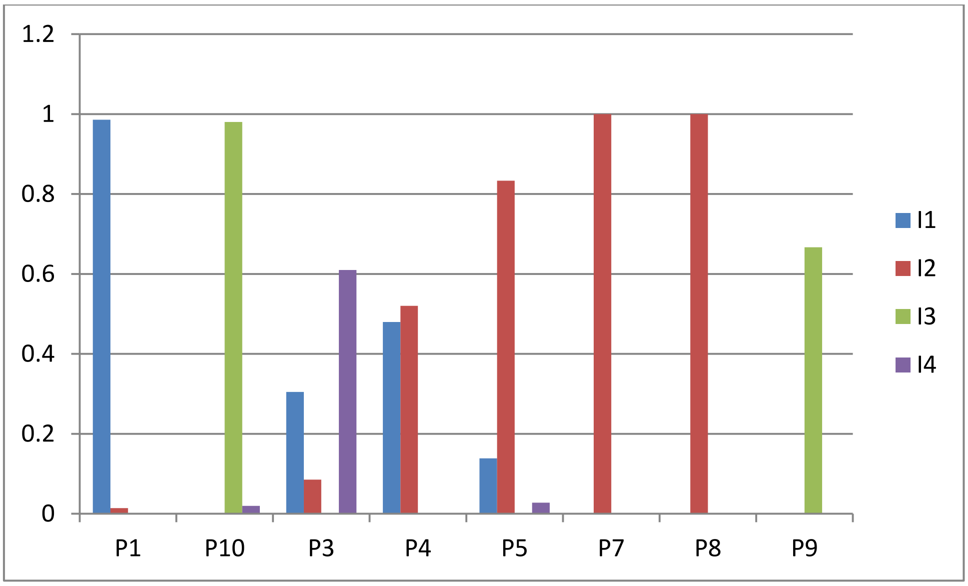

Figure 7. 10: Calculated coefficient of CRM for all production wells.

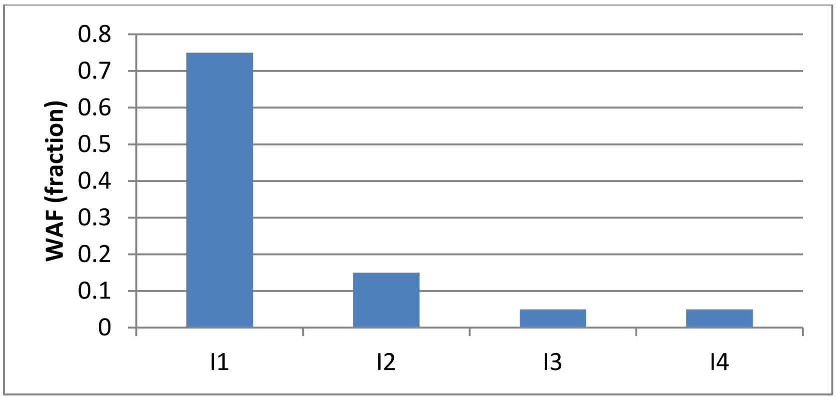

Figure 7. 11: New water allocation factor for injection wells obtained from LP-CRM technique.

7.5 Water flooding management (*WFM*)

At the end of this study all the proposed techniques for waterflood management (*RVM*, *WAM* and *PAM*) have been employed simultaneously. This scenario was called waterflooding management (*WFM*).

7.6 Results and Discussions

The following Figures compare the results of each individual management workflow together.

Injection efficiency improved a little by *PAM*, and much better improvement was obtained from *WAM* but the best performance was achieved by *RVM*.

PAM aims to increase the liquid production from the wells with good oil production and less water production. But if we increase the production from these wells we also increase the water production too. And because of this we cannot get significant improvement from *PAM*, as it will also increase water production.

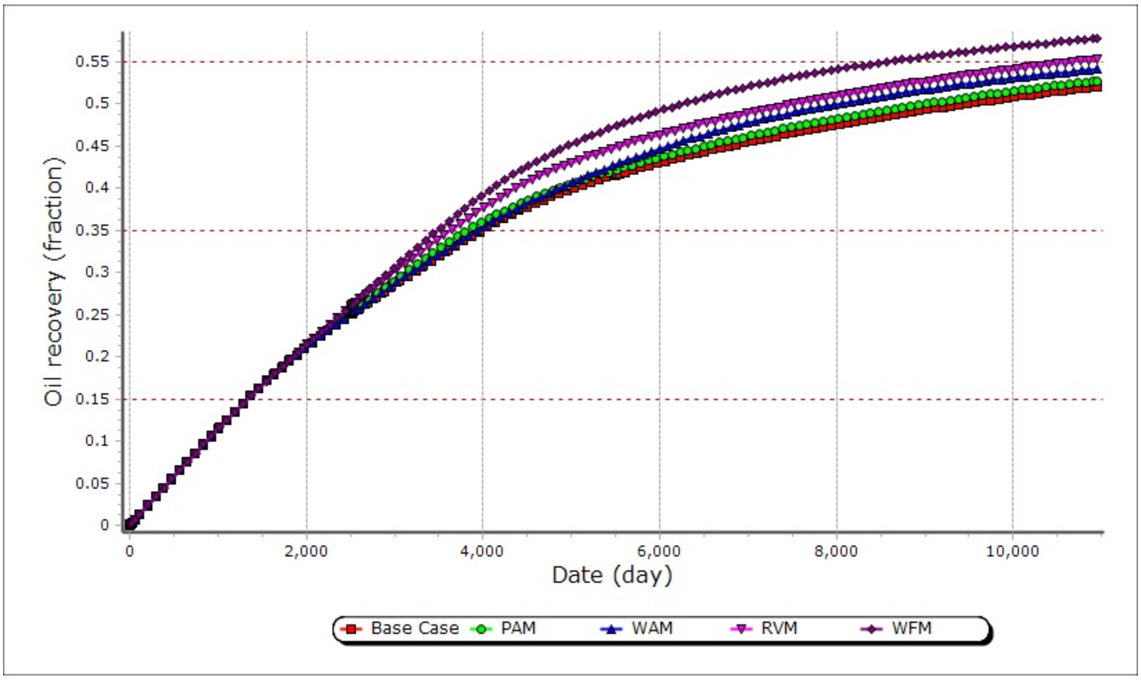

Figure 7. 12: Comparison of oil recovery obtained from different injection scenarios.

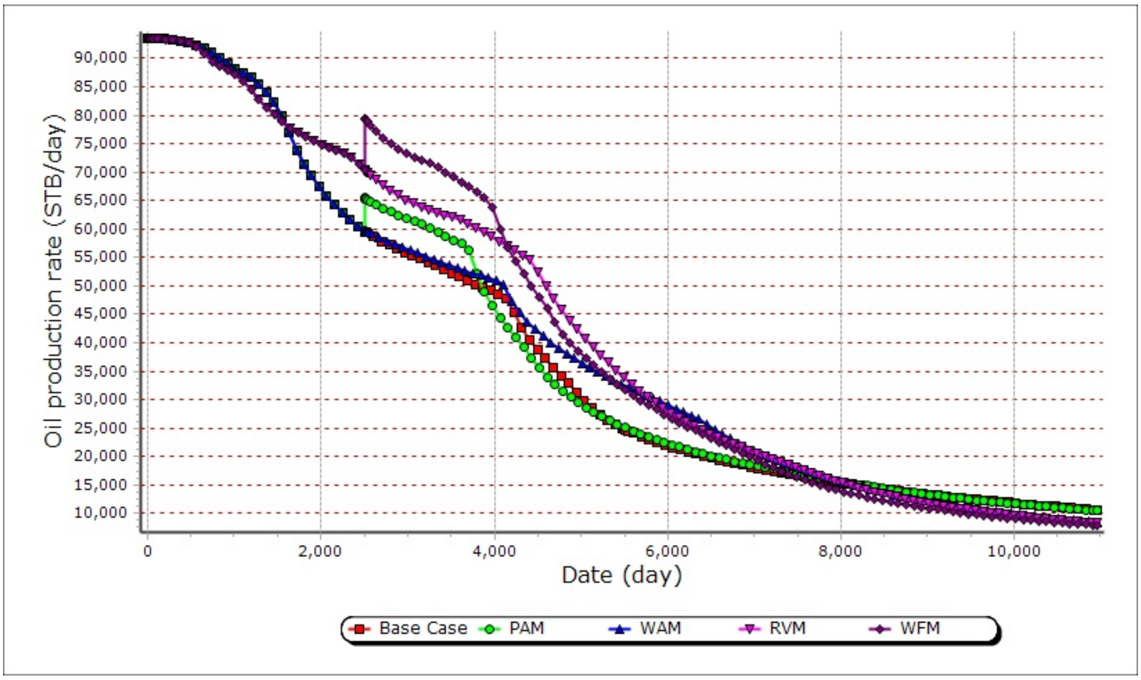

Figure 7. 13: Comparison of oil production rate obtained from different injection scenarios.

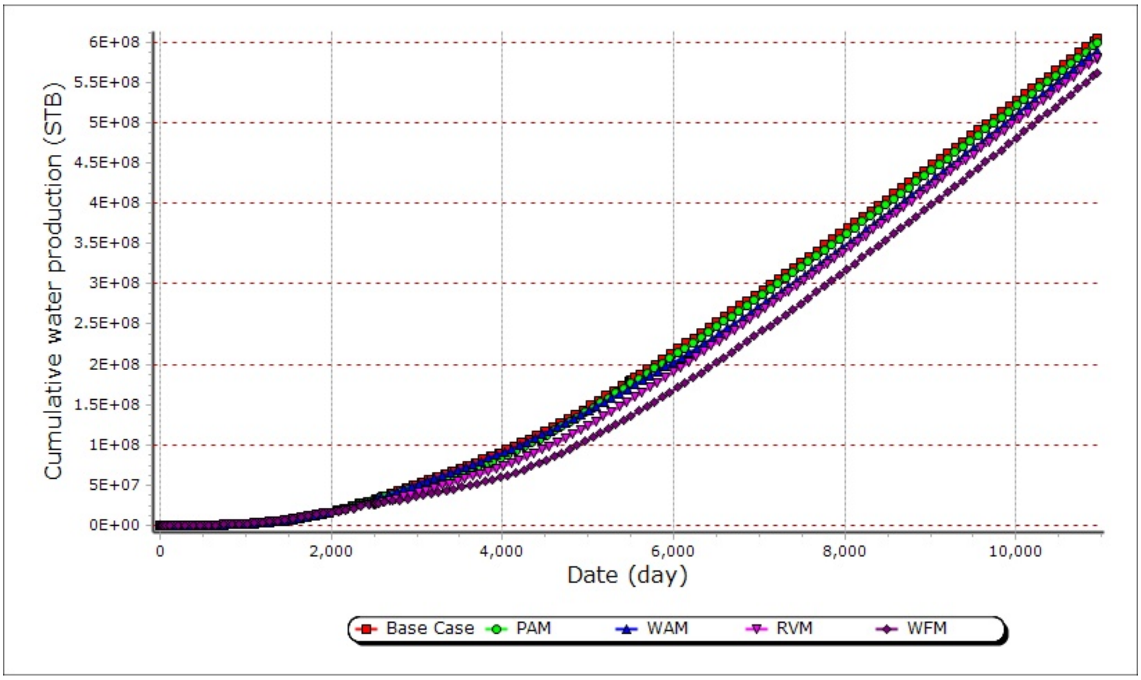

Figure 7. 14: Comparison of cumulative water production obtained from different injection scenarios.

WAM and *RVM* will affect the displacement efficiency of the water injection. This will cause more oil be pushed toward the producers without changing their production rate.

So, more oil will be produced and water production will reduce. Both *WAM* and *RVM* are improving sweep efficiency but they will act differently. *RVM* will improve waterflood movement, helping to leave less oil behind, whereas; *WAM* guides the water front in the right direction. And because of this, when we apply *WAM* and *RVM* together, we will get a very significant improvement in injection efficiency. *RVM* is like a “good driver” in a racing car and *WAM* is a “good navigator”. Finally, this study shows how effectively these simple workflows (*WFM*) can be employed to obtain a successful waterflood project in real field models.

Chapter 8– Conclusions and Recommendations

8.1 Research story

This thesis has evaluated a wide range of techniques to mitigate one of the big challenges of petroleum industry, water production. The techniques discussed for waterflooding management, all aim to reduce excessive water production. The injection and production history at a well and field level are the most common available data in any oil field, especially when nowadays we can have these data in real time with the implementation of the digital oil field and the intelligent well completion. This research aims to understand the strength and weaknesses of the existing techniques and "repackage" them to provide an optimum combination for more effective waterflood management by analysing injection and production data history.

The first part of this research reviewed, tested and compared the analytical techniques that have been previously used for analysing the injection and production. The methods studied fell in to two distinct classes: those that monitor the waterflood performance secondly, and those that determine the inter-well connectivity.

The second part of this thesis showed that an improved workflow, using the captured information from the phase one methods, could be combined to give a more effective waterflood management via combination of reservoir voidage management (*RVM*), water allocation management (*WAM*) and production allocation management (*PAM*).

Finally, a semi-analytical method was introduced in this thesis for performing *RVM*. Two approaches were defined for *WAM* and new techniques developed for *PAM*, all of which employed only the production and injection history. The results from these techniques were compared with the more advanced reservoir simulation methodologies such as gradient free optimisation. This comparison showed the reliability of the proposed techniques.

8.2 Monitoring the performance of waterflooding

8.2.1 Summary

Different techniques for monitoring the waterflood have been reviewed. These techniques were the Cut-Cum plot, the *WOR* plot, the X-plot and Hall plot. The first three can be used to forecast future oil reserve by knowing the amount of the water injected to the reservoir, while the Hall plot can be employed to monitor the performance of each individual injection wells. Finally a semi-analytical methodology was proposed for reservoir voidage management (*RVM*).

8.2.2 Conclusions

1. **The X-plot provided the most accurate estimation.**

   The results and accuracy of all three techniques were judged to be acceptable but the best future predictions were obtained from the X-plot. The X-plot has the additional advantage that it can also be used to generate an effective permeability ratio plot too. This study thus recommends X-plot for preparation of future oil recovery forecast. However, it must be mentioned that this method is only reliable for water fraction values of higher than 50%. Also, only that portion of data related to the new scheme should be used for the analysis, so that there is a significant change in the injection strategy.

   It was noted that the Cut-Cum produced better predictions than *WOR* plot, but its practical application was found to be dependant to the form of the polynomial employed to fit the data.

2. **Hall plot can be employed only to monitor the change in the conditions of injection wells. It will not give any information about the future oil reserve or recovery.**

   This study showed that Hall plot provided good information about the performance of the injection well such as injectivity index. But it could not give any information about the overall performance of the waterflood in terms of improving the oil recovery or decreasing the water production.

3. **None of the studied techniques were able to determine the optimum value for key parameters affecting the water injection efficiency.**

   The first three techniques were very good at differentiating between the performance of base case injection plan and the improved performance whan water allocation management (*WAM*) was implemented. They clearly showed that the performance of the reservoir or the injection wells had changed, improved or deteriorated. However, they were not capable of advising which change should be made to the parameters governing the injection efficiency in order to improve the overall efficiency of the injection scheme.

4. **Determination of the minimum required reservoir pressure and the time of breakthrough were showed to be two important within the Reservoir voidage management (*RVM*) parameters whose optimisation significantly can improved the waterflood's performance.**

   A high volume injection of water from the start of the production maintains reservoir pressure, but also causes early water breakthrough. This can cause a poor well outflow performance and reduces the efficiency of the water/oil displacement process. On the other hand, a reduced rate of injection of water may improve the sweep efficiency and delay the water breakthrough but will not support the reservoir pressure sufficiently to maintain the required production rate from the reservoir. Reservoir voidage management improves flooding efficiency. It will increase oil production, reduce excessive water production and require a low injection volume of water. This understanding lead to a proposed new workflow based on identifying the optimum reservoir pressure in terms of well's outflow performance and the time to water breakthrough, in order to increase the water injection efficiency.

8.3 Inter-well connectivity measurement: Statistical techniques

8.3.1 Summary

Three common statistical techniques have been tested: the Spearman rank correlation, multi-linear regression (*MLR*) and capacitive resistive model (*CRM*). A superposition approach was proposed to identify the injectors connected to each producer followed by the application the above techniques to quantify this injector/producer connectivity. Connectivity results can then be combined with the water cut of the production wells in a new, but simple, algorithm to update and manage water allocation factor.

8.3.2 Conclusions

1. **Spearman rank correlation coefficient was not able to determine inter-well connectivity.**

   Only a small improvement was obtained from water allocation management using the inter-well connectivity results delivered by this technique. It was concluded that the Spearman rank correlation coefficient is not a suitable method to define inter-well connection in the field studied. However it can identify those production wells whose performance is affected more by the entire injection than by individual injection wells.

2. **It is very important to recognise which injection wells are the connected to each production well when we measure the inter-well connectivity by the multi linear regression (*MLR*) or the capacitive resistive model (*CRM*) techniques.**

   One of the important steps in determining well connectivity is to understand the injection well production well connection. An incorrect assessment of connections will lead to a wrong quantitative measurement of the inter-well connectivity by these two methods. Superposition analysis incorporated with the location of the wells and other geological parameters can help us to ensure that the injection/production well connections are correctly assigned

3. **Inter-well connectivity measurement is a good and effective tool in water allocation management (*WAM*). WAM improved flood performance.**
   A simple new technique is proposed to define a new water allocation factor for each injection well based on the connectivity results of *MLR* and *CRM* coupled with the water cut of the production wells. This methodology allocates a greater volume of water to injection wells that are well connected to a production well with a low water cut. *WAM* improved the waterflood performance by producing more oil (improved sweep efficiency) and reducing the water production.
4. ***WAM* based on *CRM* connectivity results gave the greatest improvement**
   The injection efficiency of *WAM* based on *CRM* gave better results than *MLR* because it considers extra parameters, particularly the production well's productivity index. By calculating the time constant associated with *CRM* the

model also respects the delay time between the injector and producer. Therefore *CRM* was preferred above *MLR* for determining the inter-well connectivity.

8.4 Inter-well connectivity measurement: Artificial intelligence techniques

8.4.1 Summary

Two types of networks designed and optimised to forecast production rate by inputting the injection rate. The networks were feed-forward back propagation network (*FFBP*) and Co-active neuro-fuzzy inference system (*CANFIS*). Super position methodology used again to identify the connected injectors to each producer. In order to determine the inter-well connectivity sensitivity analysis carried on each producer. Connectivity results of these techniques combined with water cut to do *WAM*.

8.4.2 Conclusion

1. **Both Feed-forward back propagation *(FFBP)* network and fuzzy logic network (*CANFIS*) were acceptable for determining inter-well connectivity**
   Injection performance was improved by applying the *FFBP* and *CANFIS* based *WAM*. Therefore they can be employed to determine inter-well connectivity. However *FFBP* was better than *CANIS*
2. **Statistical techniques (*MLR* and *CRM*) were better than *FFBP*.**
   FFBP based *WAM* improved oil production and reduce the production of water. However *MLR* and *CRM* were even better than *FFBP* at measuring the inter-well connectivity as well as employing a simpler calculation workflow. Inter-well connectivity can be obtained directly from *MLR* and *CRM* while a sensitivity analysis was required to determine the connectivity with *FFBP*. *CRM* is thus recommended for calculating the inter-well connectivity.

8.5 New development in water allocation management (*WAM*) and production allocation management (*PAM*)

8.5.1 Summary

The study described above showed that, unsurprisingly, the production well's water cut can be altered by changing the water allocated to the producers which were highly affected by injectors. Also it did not necessarily fully respect the previous performance of the well. A search was thus made for a better parameter that describes the performance of the production wells. This parameter should separate a good producer from a bad producer more clearly. New parameters that were chosen for were: cumulative water cut (*CWC*), relative oil production ratio (*OPR*) and oil production index (*OPI*). In all cases *WAM* based on *CRM* connectivity results was used to test the newly defined parameters.

New techniques have also been developed for production allocation management with the above parameters (*PAM*). Finally production and water allocation management (W*&P AM*) have been done simultaneously.

8.5.2 Conclusions

1. ***CWC* can be used for long-term *WAM* while *WC* will be better for short term *WAM*.**

   Both *WC* and *CWC* gave improved results. But at the beginning *WC* worked better than *CWC* at the early time, but, as expected *CWC* performed above WC after longer production intervals. It is also preferred since the allocation factor obtained from *CWC* was more stable to changes in performance of the production wells than the value obtained from *WC*. *WC* thus is the preferred short-term parameter while *CWC* should be applied for longer time management.

2. **New *WAM* methods employing *OPR* and *OPI* provide significant improvements compared to *WC* and *CWC* with *OPI* giving slightly better results.**

   WAM methods employing the newly defined parameters *OPI* and *OPR* were very effective in improving injection efficiency. The power of *OPR* and *OPI* is that they provide a greater separation of good and bad production wells since they can assign a negative value to bad producers. This greater differentiation ensured that less water was allocated to the injectors connected to the producers with highest water production with more water went to support production wells with a higher level of oil production. More oil and less water production resulted. *OPI* was more effective at this since it gives more difference between “good” and “bad” producer.

3. **A clear differentiation between good producers and bad producers is very important for efficient *WAM*.**

WAM based on *OPI* and *OPR* gives a greater differentiation more between "good" production wells and "bad" production wells. This allows application of more "punishment" to the injectors supporting "bad" production wells, delivering the greatest improvement in the waterflood performance

4. **Improvement from *WAM* is better from *PAM*.**
   WAM is more effective at improving the long-term waterflood efficiency (more cumulative oil and less cumulative water production) than *PAM*. *PAM* delivers the greater short-term oil production while the *oil* production improvement only builds up slowly in the case of *WAM*. This is because there will be a considerable lag time before increase the production rate as a result of changes to the injection strategies (the reservoir acts at a much longer time scale than a single well). The reduction in water production was bigger than the increase in oil production for *WAM* when compared to *PAM*.
5. **The combination of *WAM* and *PAM* (*W&P AM*) provided best performance.**
   W&P AM improved significantly the performance of the analysed reservoir models. It was much more effective at increasing the cumulative oil production (the amount of improvement was two times more than *WAM* and *PAM*) rather than decreasing water production. This is because *PAM* is better in improving oil production rather than decreasing water production.

8.6 Water allocation optimisation

8.6.1 Summary

All the above studies performed *WAM* based on the inter-well connectivity results and monitoring the resulting production well performance. An alternative approach is to use water allocation factors based on a future forecast of the production rates following the changes in the injector's injection rates. Such techniques use the injection rates as an input to estimate future liquid production rate in the production wells (*MLR* & *CRM*) coupled with an optimiser engine to optimise the injection rate. Reviewing *MLR* and *CRM* shows that both of them, along with their constraints and the objective function are linear. Therefore linear programming (*LP*) based optimiser can then be used to find the optimum allocation factor for the injectors. Two objective functions were considered; maximising daily rate and maximising the cumulative oil production.

Finally, the results from the *LP-CRM* study compared with those from the combination of Genetic algorithm (*GA*) optimiser with reservoir simulator.

8.6.2 Conclusions

1. **The second approach of WAM (*LP-CRM*) was better than the previous methodology based on connectivity measurement (*OPI-CRM)*.**

   LP-CRM guided *WAM* improved the overall water injection performance even better than the previous methodology based on injecting more to the injectors connected to good producers (*OPI-CRM*). Allocation management based on future performance estimation is thus better than allocation based on past performance.
2. ***LP-CRM* was very sensitive to the defined objective function.**

   Objective function has a significant effect on the determination of the optimum solutions. Maximisation of daily oil rate is thus a better objective function for short term optimisation while maximising the cumulative oil production was better for long-term allocation management.
3. ***LP-CRM* is recommended as an effective tool for *WAM*.**

   A marginal but not significant improvement was obtained from a GA-simulation combination compared to an *LP-CRM* based allocation management. The computation time of the GA-simulation was 24 hours, compared to less than 15 minutes for the *LP-CRM*. The *LP-CRM* thus is recommended as an effective and simple methodology for water allocation management especially when a reliable reservoir model is not available, or for a very large reservoir models containing many of wells when the required computational time is not practical. LP-CRM prediction will also be very useful even when powerful computers with sophisticated reservoir models are available since it can be used to define the first guess for the GA-reservoir simulator to reduce the computing time.

8.7 Water flooding management: new case study:

8.7.1 Summary

The proposed techniques for *PAM*, *WAM* and *RVM* were then applied to a more complex real field model to evaluate the added value of waterflood management.

8.7.2 Conclusions

1. ***RVM* is a "good driver" and *WAM* is a "good navigator".**

PAM aims to increase the liquid production from the wells with good oil production and less water production. But if we increase the production from these wells we also increase the water production too. And because of this we can not get significant improvement from *PAM* as it will also increase the water production. *WAM* and *RVM* will affect the displacement efficiency of the water injection. This will cause more oil be pushed toward the producers without changing their production rate. So, more oil will be produced water production will reduce. *WAM* and *RVM* will both are improving sweep efficiency but they will act differently. *RVM* will improve waterflood movement helping to leave less oil behind on the other hand; *WAM* put the water front in the right direction. And because of this when we apply *WAM* and *RVM* together we will get very significant improvement in injection efficiency

8.8 Recommendations for future works:

1. New techniques have been developed for *RVM*, *WAM* and *PAM* based on production and injection rate data. These techniques can be used in principle to design a closed loop water injection management based on intelligent completion. So a study of the feasibility of designing such a closed loop water allocation management and the added value of it can be a good area for further research.
2. The proposed techniques were based on exercising on reservoir models. So it is strongly recommended to validate the proposed techniques in real forecasting cases where real field injection and production history are available.

3. In this study we tried to calculate the connection between injection and production wells at the well scale (whole reservoir section). But intelligent completion will provide us with monitoring and control at a layer scale; hence evaluation of new inter-well connectivity at each individual can be incorporated in the overall waterflood strategy at each individual layer.

4. The role of an active aquifer was not considered in this study. Allocation management in presence of active aquifer in the reservoir is another area for study.
5. This research assumed that no new production or injection wells were drilled during the life of the field. A systematic Study of the effect of new drilling on well connectivity can be another useful subject for water allocation management.
6. The effect of different injection schemes on determination of well connectivity can be another subject for further research such as injection with constant bottom-hole pressure or constant surface injection rate.
7. There are other techniques that have the potential to be employed to determine inter-well connectivity. One of these possible methods could be advanced interpolation techniques such as kriging.